COLLECTED COMMENTARY RELATING TO THE FLOW OF WATER AND DREDGED SOLIDS THROUGH HYDRAULIC PIPELINES

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by

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PREFACE

During the past year that the author has had the privilege of undertaking postgraduate study at Princeton, the field of hydraulic dredging has presented itself repeatedly in the literature of waterfront construction and harbor improvement as a subject of challenging magnitude. It is partly a result of the author's ignorance of the subject and partly due to his interest in the large economies that can result from the use of hydraulic fills, that a study of the field of hydraulic dredging was undertaken.

An examination of hydraulic dredging, its problems and those areas needing applied study led the author to write several firms engaged in this field. To each addressee was directed the question, "What subject or subjects in the broad field of dredging do you feel is in need of study and research?" In reply to this query, Mr. DeWitt D. Barlow, Jr. of the Atlantic, Gulf and Pacific Company listed three subject areas relating to dredge pipelines as follows:

a. Investigation of the transition point (or range) between colloidal solutions and mixtures of water and solids.

b. A study of the relation of friction losses to percent solids in the mixture; or the relation of friction losses to particle size.

c. A study of the most economical pipeline velocity.

The author is very much indebted to Mr. Barlow, for it is upon the three proposals listed above that this thesis is centered and, concerning which, a concerted effort has been made to develop a single source of information. Of great benefit and inspiration to the author have been the assistance and encouragement of Dr. R. H. Wilhelm, Chairman of the Department of Chemical Engineering; Vice Admiral W. M. Angas, Chairman of the Department of Civil Engineering; Professor N. J. Sollenberger and Professor J. L. Green both of the Department of Civil Engineering. In particular, the personal encouragement of Professor Sollenberger, as thesis advisor, has been most stimulating and helpful. Last but not least, the humble gratitude of the author is expressed to his wife, Anne, who suffered through the role of student wife once more, to the end, that the author could complete his studies.

The author has applied to his subject matter the knowledge of his background as a Mechanical Engineer (Duke University, 1945), a Civil Engineer (Rensselaer Polytechnic Institute, 1952), a Naval Officer in the Civil Engineer Corps with 16 years of varied and extensive service in all phases of engineering, and as a registered professional engineer (N.Y. State) However, it is with limited previous specific knowledge of his subject and with a sincere appreciation of his shortcomings that the author has set forth his considerations and evaluations of the work of others in this field and allied fields covering fluid transport criteria and phenomena.

SUMMARY

A study has been conducted of the published information available relating to the flow of water and dredged solids in hydraulic pipelines. The author presents a colligation of data and information on three specific subjects.

a. Transition point or range between colloidal mixtures and water-solids mixes.

b. Relation of friction loss to particle size or concentration.

c. Economic pipeline velocity.

This threefold division of transport phenomena is treated broadly under a development of theoretical aspects, an extensive reference survey, and an analysis of the various approaches and considerations advanced by other writers.

It is concluded that for water-solids flow:

a. Little published data is available on the transition point or range between colloidal mixture and water-solids mixes and, hence, no distinct demarcation can be stated. However, there appears to be little to commend study of this relationship as an important flow characteristic.

b. Friction loss seems best related to concentration rather than particle size.

c. The most economical pipeline velocity is that which corresponds to incipient deposition.

The author recommends that much greater effort be made in applied research and evaluation particularly in the evaluation of the currently published theories of solid-liquid flow. Full scale tests of dredge equipment under actual operating conditions with the investigation of theoretical relationships as a central aim are strongly needed. Much of the currently available theoretical knowledge awaits the proof or disproof of such actual tests.

The author contends that utilization of existing theoretical data and formulae for design and operating procedure depends on the outcome of future experimentation and investigation. At present, there is little in the theoretical that could replace the current "rule of thumb" operation of the experienced dredge operator.

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NOTATION

a	- Distance measured from bed of channel where C_a is measured	L
A	- Area	L^2
A_1, A_2, A_3	- Constants	--
C	- Local concentration in per cent by weight	--
C_1	- Relative absolute volume of sediment	--
C_a	- Local concentration in per cent by weight at distance a from the bed of channel	--
C_T	- Total sediment load concentration in per cent by weight	--
C_x	- Drag coefficient for a given sediment size	--
d	- Median diameter of sediment in mm	L
D	- Diameter of the pipe	L
D_1	- Depth of flow	L
E	- Energy required per foot of pipe per pound of sediment transported per second	LF/L/F/T
F_D	- Drag force	F
f	- Darcy-Weisbach resistance coefficient with or without sediment present	--
f_1	- That component of f for sediment-laden flow, which is contributed by clear water flow	--
f_2	- That component of f for sediment-laden flow, which is contributed by total sediment load carried; $f = f_1 + f_2$	--
Fr	- Froude Number $\left\lvert \frac{V^2}{gD} \right\rvert$	--
g	- Gravitational acceleration	L/T^2
I	- Intercept from d/D versus P plot	--

J	- Hydraulic gradient along the pipe length with or without sediment	--
J_e	- Hydraulic gradient along the pipe length with clear water	--
K	- Karman constant	--
KE	- Kinetic energy per unit volume	$(F)(L)/L^3$
k	- Nikuradse equivalent uniform sand diameter	L
K_1, K_2, K_3	- Constants	--
ℓ	- Mixing length in the theory of turbulence	L
L	- Length of pipe	L
M	- Rate of lateral transport	F/T
m	- Constant	--
n	- Constant	--
p	- Pressure	F/L^2
P	- Intercept on the $Re\sqrt{f}$ versus C_T plot	--
Q	- Discharge of clear water or water sediment mixture	L^3/T
q	- Ratio of solids to air by weight	--
r	- Distance from pipe center line to point of determination	L
R	- Radius of pipe or tube ($2R = D$)	L
Re	- Reynolds Number	--
s	- Slope of energy gradient, slope of water surface for open channel flow, or slope of pipe	--
S_1	- Reciprocal of slope of plot of P versus d/D	--
S_F	- Shape factor for sediment	--
t	- Time	T

u, v, w	- Mean velocity at a given point in the x, y and z directions respectively	L/T
u', v', w'	- Velocity fluctuations in x, y, and z directions, respectively	L/T
V	- Mean velocity over the total area of pipe	L/T
V_L	- Limit deposit velocity	L/T
w	- Settling velocity of the sediment particle	L/T
W	- Rate of sediment transport	m/T
y	- Depth of flow	L
y_s	- Depth of sand deposit	L
Z and Z_1	- Constants	--
β	- Factor of proportionality	--
γ_s	- Unit weight of sediment	F/L^3
γ_m	- Unit weight of mixture	F/L^3
γ_w	- Unit weight of water	F/L^3
ε	- Dynamic coefficient of turbulent viscosity (eddy viscosity)	L^2/T
ε_m	- Momentum exchange coefficient	L^2/T
ε_s	- Sediment exchange coefficient	L^2/T
Φ	- Dimensionless function	--
η	- Coefficient of turbulent viscosity or "eddy" viscosity	FT/L^2
λ	- Longitudinal spacing between roughness elements	L
μ	- Coefficient of viscosity	FT/L^2
ν	- Dynamic viscosity	L^2/T
ρ	- Mass density of mixture	$\frac{FT^2/L}{L^3}$

ρ_a	- Mass density of air	$\frac{FT^2/L}{L^3}$
ρ_s	- Mass density of sediment	$\frac{FT^2/L}{L^3}$
ρ_w	- Mass density of fluid	$\frac{FT^2/L}{L^3}$
σ_g	- Geometric standard deviation	--
τ	- Intensity of shear	F/L^2
ϕ	- Variable characterizing boundary form	--
ψ	- Function of	--
Δp	- Pressure differential	F/L^2

CHAPTER I. INTRODUCTION

Purpose

The purpose of this thesis is primarily the collection of data and written material on the movement of dredged solids through pipelines. Particular emphasis is to be placed on the following within the limits of available source material and knowledge:

a. Transition point or range between a colloidal mixture and water-solids mixture.

b. Relation of friction loss to particle size or concentration.

c. Study of the economic pipeline velocity.

It is fundamental, then, to the purpose of this thesis to centralize and present, to the greatest possible extent, all of the available information on the subject matter.

In particular, this thesis will present a reference study of the subject, an analysis of most of the current methods or approaches to the resolution of data in theory and practice, and the author's summary and recommendations as to future requirements, research, and further evaluation.

Scope

The general endeavor of the author has been to limit this writing to information concerning the threefold purpose aforementioned. However, instances do occur where added

information has been recorded in the interest of fully obtaining clarity of the subject matter. The attempt has been made to gather related information from other fields with similar problems. Therefore, the reference study contains much of the data published by mechanical engineers, chemical engineers, aeronautical engineers, sedimentologists, and many others, as well as civil engineers. In each case, the intent is to insure that data or information analogous to hydraulic dredging is presented.

In hydraulic dredge pipelines, a wide range of physical characteristics is observed. For example, solids can range from colloidal particles to large boulders 27 inches in maximum dimension; pipelines are normally smooth circular conduits of which 30 inch diameter sizes are not uncommon; pipeline velocities range up to 36 feet per second and more; concentrations average about 15% by weight, but are apt to vary considerably depending on the efficiency of the leverman. The scope, then, of this thesis is on the one hand quite broad if all of the individual physical criteria which enter the process of transportation of dredged solids are considered, and quite narrow in the sense that the process is only a small part of the broad field of two phase transport phenomena.

Dredging and the Hydraulic Pipeline

The first hydraulic dredge was built in the United States in 1872. From then until the present dredges and dredging

operations have been developed largely through the teachings of field experience. "Rule of thumb" is the common basis for practically all dredging operations. In very few fields, particularly one of the age of dredging, does one encounter such a lack of published theory regarding design and operational problems. However, this is not too surprising for when one examines the developments that have been made in fluid mechanics, it is quickly noted that much of the current understanding of fluids and fluid phenomena has been developed in the last 30 years. Still, the author was quite amazed to find that relativel little theory has been developed and that which exists has not been fully tested, evaluated, or, more important, applied.

It is significant that the cost of just the annual maintenance dredging in the United States for harbor and river channel work is a multimillion dollar business that keeps a large fleet of dredges belonging to the U. S. Army Engineers and a sizeable number of private dredges engaged in full time operation. Also of note is the underlying fact that fill material or excavated material can be moved by dredging and hydraulic pipeline at a cost which is considerably cheaper than any other presently known method of earth moving, the only proviso being the availability of water. A recent example of this is the Dutch firm that successfully completed a contract for waterfront construction that involved, among other things, the excavation of 40 feet of peat from a 300 acre site in England. The job

was accomplished at a reduced cost by dismantling a dredge in the Netherlands, shipping it to England, reassembling it on the construction site, flooding the area and, finally, dredging the peat. The economics of dredging, then, are such that any research which produces a small improvement can, in fact, effect large savings in costs.

It is the intent of this thesis, however, to treat only a portion of the subject of dredging. This portion may be roughly described as the operation and functioning of the hydraulic dredge pipeline. At this point it is well to state that, for the purposes of this thesis, the words, sediment, dredged solids, dredged material, or solids, are all used interchangeably to describe broadly the discrete solid particles which may be transported by a given fluid. In this small sector of dredge operations, pipeline flow of water and solids, the author has found the work of others on recent transportation of coal and water mixtures by pipeline extremely helpful, as is the work of sedimentologists in the literature of sediment transport on rivers and alluvial streams. Papers treating the subjects of irrigation, drainage, sewerage, transport of minerals, ores, and slurries, as well as many other industrial processes dealing with two phase flow have been extremely valuable as source material for this thesis. Some authors have attempted to bring much of this data together, however, it is surprising that much greater effort has not been expended in this regard. The author is firmly convinced that given ample time a more

thorough reference study than that which is presented in this thesis, would be of great benefit to the dredging industry. Such a study would undoubtedly provide a firmer scientific basis for further research and understanding of physical processes which are today only vaguely understood.

CHAPTER II. THEORETICAL BACKGROUND

A study of the subject of this thesis would not be complete without adequate review of the field of fluid mechanics. With a guide from the work of Bird, Stewart, and Lightfoot (B-15), a summary of the basic fundamentals and current knowledge related to the subject is present.

Viscosity

The chief physical property with which one deals with in fluid dynamics is the coefficient of viscosity. Newton's law of viscosity states that the viscosity of a fluid (gas or liquid) may be defined as a constant of proportionality between the shear force per unit area and the negative gradient of the local velocity, and written as

$$\tau = -\mu \frac{du}{dy} \qquad (1)$$

Fluids which follow Newton's law of viscosity are known as Newtonian fluids and are characterized by a linear relationship, passing through the origin, when the shear stress, τ, is plotted against the velocity gradient, see Figure (1). However, there are many fluids which do not follow the fundamental relationship of equation (1) and these are referred to as Non-Newtonian fluids. Of the many non-Newtonians, three provide useful classification and are standard terminology in the field. See Figure (1).

The Bingham Plastic is a material which does not yield until a finite shear stress τ_o has been attained; after this

FIG. I — SUMMARY OF RHEOLOGICAL BEHAVIOR OF NEWTONIAN FLUIDS & OF SOME NON-NEWTONIAN MODELS (B-15)

shear stress is reached the fluid flows like a Newtonian fluid with viscosity μ_o. This can be written as

$$\tau = \pm \tau_o - \mu_o \frac{du}{dy} \tag{2}$$

The Pseudoplastic materials are those which seem to get less viscous as they are sheared, whereas the Dilatant materials get more viscous as they are sheared. Many substances of these types may be represented approximately by a relation known as the power function model which may be written as

$$\tau = -m \left| \frac{du}{dy} \right|^{n-1} \frac{du}{dy} \tag{3}$$

in which m and n are constants, where n represents the deviation from Newtonian behavior and m may be likened unto viscosity. Much information is lacking about non-Newtonian materials and there is considerable need for research in this regard. Of interest to the subject at hand, however, are two materials that Metzner (M-15) reports data on the parameters m and n for the power function model.

23.3% Illinois yellow clay in water, m = 0.116 and n = 0.229

54.3% Cement rocks in water, m = 0.0523 and n = 0.153

Fluid Flow

Fluid flow may be characterized as steady or unsteady, uniform or non-uniform, and laminar or turbulent. If, at any typical point, the velocity of successive fluid elements is the same in both magnitude and direction at successive instants, the

flow is said to be steady. If, at any instant, the velocity of successive fluid elements along a typical stream tube is the same in both magnitude and direction, the flow is said to be uniform.

Flow in which the shear is characterized by basic relationship expressed in equations (1) and (2) has come to be known as <u>Laminar</u> or <u>Viscous</u> flow. Each such successive fluid layer or laminae remain distinct from one another and, thus, laminar flow can be graphically depicted as a series of non-intersecting flow lines. The basic relationship for laminar flow is known as the <u>Hagen-Poiseuille</u> law and is based on laminar incompressible flow in a tube. It is expressed as

$$Q = \frac{\pi \Delta p R^4}{8 \mu L} \qquad (4)$$

Expressions for the average velocity, maximum velocity and drag force can easily be developed from equation (4). In addition, the expression can be modified to provide for other boundary shapes and for fluid areas. The assumptions which are implied in the development of the Hagen-Poiseuille law are:

a. Laminar flow - Re less than 2×10^3.

b. End effects neglected - the deviation from parabolic velocity profile and tube entry and tube exit is not important if the tube is more than 50 diameters in length.

c. Incompressible flow - the density ρ is constant.

d. Newtonian fluid.

e. Steady state flow (time independent).

f. No slip at wall.

g. Gravity effects are neglicted.

At this point it is well to define two groups of dimensionless numbers which are used repeatedly in engineering studies.

$$Re = \left| \frac{DV\rho}{\mu} \right| = \text{Reynolds number} \qquad (5)$$

$$Fr = \left| \frac{V^2}{gD} \right| = \text{Froude number} \qquad (6)$$

Reynolds number is characteristic of the type of fluid flow and is a measure of the relative importance of the viscous forces in the system. The Froude number is a measure of the relative importance of gravity forces in the system.

Most laminar flow problems can be solved by application of the equations of continuity, motion, state and viscosity as required. The <u>equation of continuity</u> for a region $\Delta x, \Delta y, \Delta z$, fixed in space, through which a fluid is flowing can be expressed as follows:

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial}{\partial x} \rho u + \frac{\partial}{\partial y} v + \frac{\partial}{\partial z} \rho w \right) \qquad (7)$$

This equation simply describes the change of density with respect to time at a fixed point in space, this change resulting from the changes in the various components of the mass velocity vector. More to the point, the equation states that there is a

conservation of mass in any differential element of volume within the fluid. For an incompressible fluid equation (7) reduces to the form of

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{w}{\partial z} = 0 \qquad (8)$$

The equation of motion usually takes some form of the famous Navier Stokes equations which are listed here for constant ρ and μ. The three equations are of the form of

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} +$$

$$\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x \qquad (9)$$

The three equations of equation (9) obtain from a momentum balance. A general statement of a momentum balance is that within a small element of volume fixed in space the momentum per unit volume changes because of (a) the transport of momentum into and out of the fluid element by virtue of the bulk fluid flow, (b) the difference in pressure on opposite sides of the volume element, (c) the difference in shear stress on opposite sides of the volume element, and (d) the gravity force acting on the fluid contained within the volume element.

When μ is set equal to zero, equations (9) result in the Eulerian equations of acceleration. Dimensional analysis of

equations of continuity and motion lead to the formation of Re and Fr. Also, it can be generally stated that application of the previous equations to laminar flow provides an exact description of the flow behavior, the only limitation placed on the investigator is the complexity of the mathematical solutions.

Turbulence

Once the fluid reaches a flow characteristic $Re \approx 2 \times 10^3$, the laminar flow becomes unstable, eddies generated in the initial zone of instability spread rapidly through the fluid, and the additional disturbances which they themselves produce eventually cause a disruption of the entire flow pattern. The result is fluid turbulence. The major distinction between turbule and laminar flow therefore lies in the existence of a complex secondary motion superimposed upon the primary motion of translation (R-9). Since the average size of the turbulent eddies is much larger than the length of the mean force paths of the molecul making up the fluid, the fundamental equations developed for lamin flow can be applied to turbulent flow on a quasi-empirical basis. However, the strict application of the laminar flow equations would give instantaneous velocities and pressures, which for the random motion of turbulent flow, would give greatly varying results. To avoid this difficulty the use of time-averaged or temporal mean quantities are used in dealing with turbulent flow. This procedure will lead to the turbulent momentum flux (or shear stress) components which are often referred to as the "Reynold's stresses".

Fluid flow does not change abruptly from laminar to turbulent flow, in fact, laminar flow may persist in a circular tube above Re = 2×10^3 by keeping the flow system free from vibrations or by keeping the tube surface free from scratches. Generally, any normal amount of surface or vibrational disturbances will cause the well-ordered laminar motion to give way to the random motion of the turbulent flow. This is true for the bulk of motion in the tube, however, right at the surface of the tube there usually remains a thin layer of fluid moving in laminar flow called the laminar sub-layer. The transition from laminar sub-layer to the zone of fully developed turbulence is not discontinuous and the region of transition between the two is called the "buffer zone". Thus, in considering turbulent flow systems, one is dealing with a complex system involving three regions which exhibit more or less distinct behavior (a) the <u>laminar</u> sub-layer, in which Newton's law of viscosity describes the flow, (b) the <u>buffer zone</u> in which the laminar and turbulent effects are both of importance, and (c) the <u>region of fully developed turbulence</u> in which purely laminar effects are of negligible importance. See Figure (2).

From the Hagen-Poiseuille law, equation (4), it can be established that for <u>laminar flow</u> in a circular tube, the velocity distribution is parabolic. u and V are given by:

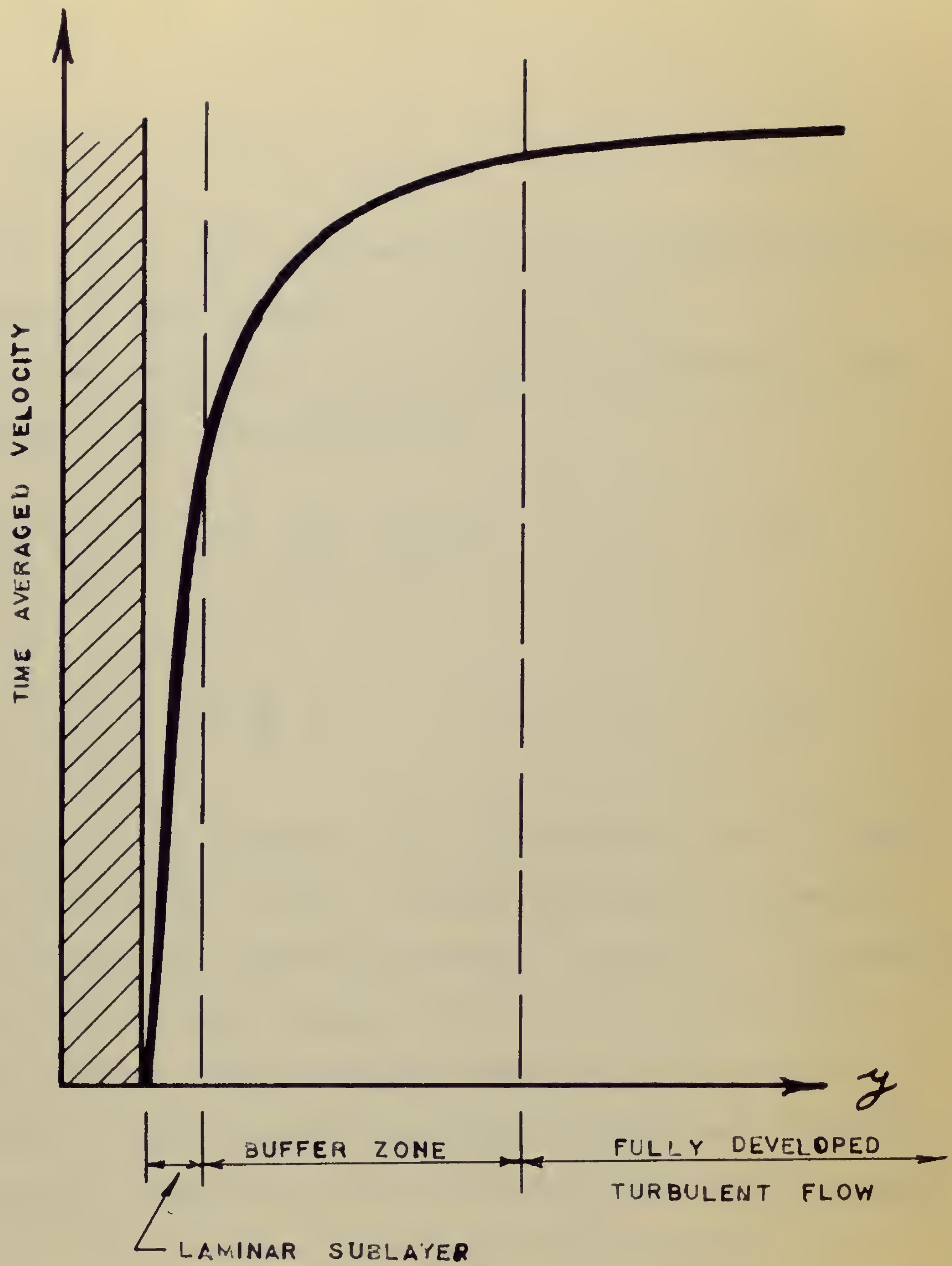

FIG. 2 — VELOCITY DISTRIBUTION FOR TURBULENT FLOW IN TUBES—REGION NEAR THE WALL

$$\frac{u}{u_{max}} = \left[1 - \left(\frac{r}{R}\right)^2\right] \qquad (10)$$

$$\frac{V}{u_{max}} = \frac{1}{2} \qquad (11)$$

Also, it is seen that the pressure drop is exactly proportional to the volume rate of flow.

For turbulent flow it may be shown experimentally that $\bar{u}$ and $\bar{V}$ are given approximately by

$$\frac{\bar{u}}{\bar{u}_{max}} \approx \left[1 - \frac{r}{R}\right]^{\frac{1}{7}} \qquad (12)$$

$$\frac{\bar{V}}{\bar{u}_{max}} \approx \frac{4}{5} \qquad (13)$$

The corresponding pressure drop is proportional to 7/4 power of the volume rate of flow. Figure (3) shows a comparison of the laminar and turbulent velocity profiles. To be noted is that the empirical formula, equation (12), does not properly account for the regions near the tube surface. It is also evident that the empirical formula is of no value if one needs information about the drag force, which depends on the velocity gradient evaluated at the surfaces.

Reynolds sought to simplify the application of the laminar flow equations of motion and continuity (Equations 8 and 9) by substituting for every instantaneous velocity

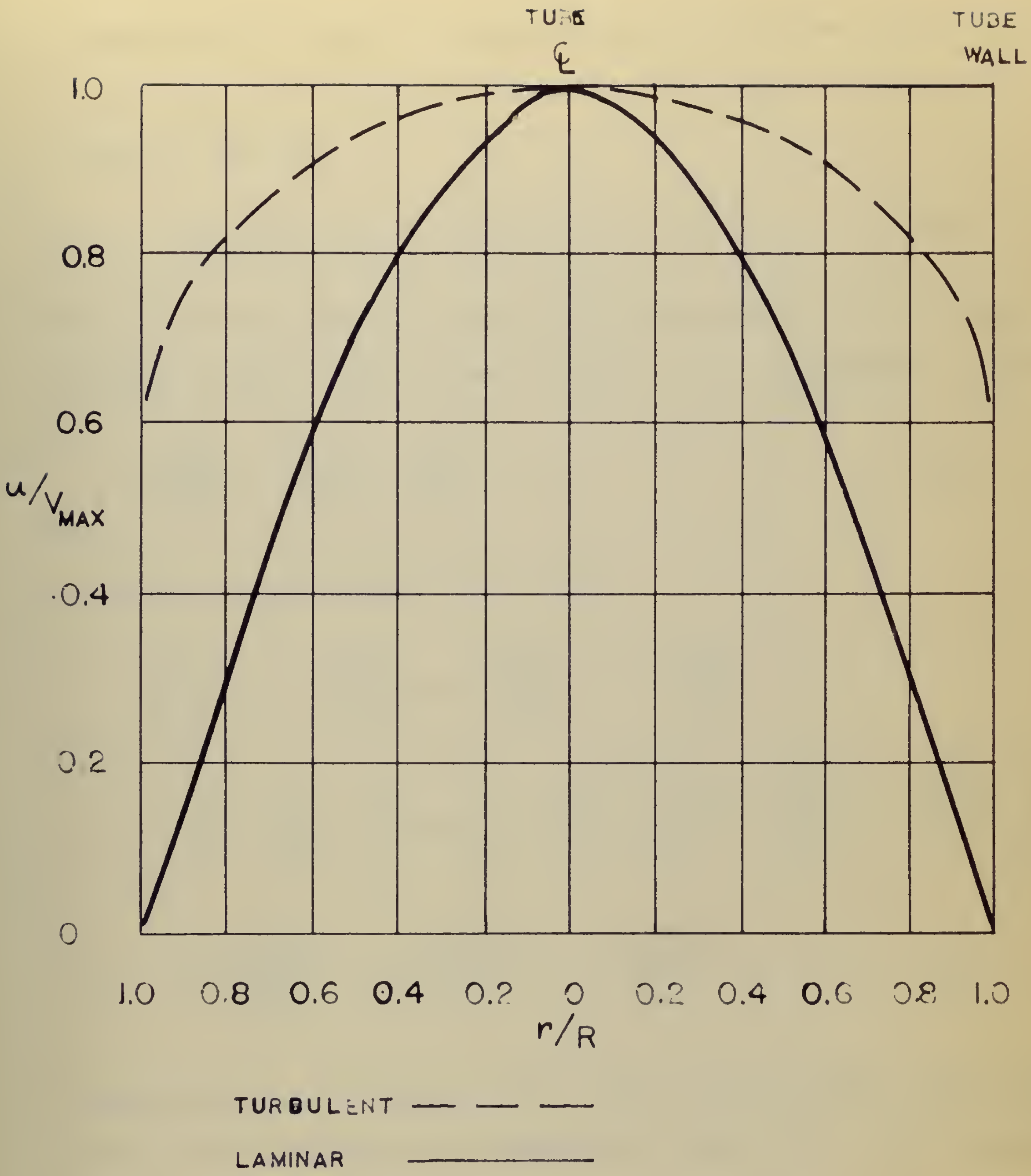

FIG. 3 - QUALITATIVE COMPARISON OF LAMINAR & TURBULENT VELOCITY DISTRIBUTIONS (B-15)

component the sum of a temporal mean component (denoted by a bar) and the momentary departure therefrom (denoted by a prime); that is,

$$u = \bar{u} + u', \quad v = \bar{v} + v', \quad w = \bar{w} + w' \tag{14}$$

Use of these quantities and the elimination of all terms having a mean value of zero results in a <u>continuity</u> equation

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \tag{15}$$

and three equations of the forms

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} =$$

$$-\frac{1}{\rho}\frac{\partial}{\partial x}(\bar{p} + \gamma z) + \frac{\mu}{\rho}\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2}\right)$$

$$-\frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z} \tag{16}$$

Examination of equations (15) and (16) indicates that the effect of turbulence is embodied in the three mean products of the components of fluctuation.

From the above it can be shown that

$$\bar{\tau} = \mu\frac{d\bar{u}}{dy} - \rho\,\overline{u'v'} \tag{17}$$

for steady, uniform turbulent flow in the x direction. (R-9)

The contribution of the turbulent motion to the local shear depends on the magnitude of the mean product $\overline{u'v'}$. Since, according to the definition of $\bar{u}$ the means of u' and v' themselves must necessarily be zero, it is seen that the mean product can have a finite magnitude only if the magnitude of u' is to some extent related to the magnitude of v'. Perfect correlation does not exist, however, measurements of flow in a pipe with appreciable shear invariably results in a finite correlation of sorts.

Beginning with the above analysis, several attempts have been made to deduce expressions for the Reynolds stresses. All are empirical in nature and most have been attacked for one reason or another. However, since much current theory of hydraulics embraces one or more of these approaches, several are outlined below.

<u>Boussinesq Eddy Viscosity Expression</u> (R-9)

One of the earliest was proposed by Boussinesq who drew a parallel between the molecular motion of laminar flow and the molar or eddy motion of turbulent flow. The molecular viscosity μ of a fluid is known to depend upon the density, the velocity, and the mean free path of the molecules. Therefore, it should be possible to introduce a molar or eddy viscosity, η , which depends on the density, velocity, and

size of the eddies, such that (R-9)

$$\bar{\tau} = (\mu + \eta) \frac{d\bar{u}}{dy} \qquad (18)$$

This equation has little to recommend it because η depends strongly on position.

<u>Prandtl's Mixing Length Expression</u> (B-15, R-9)

By assuming that eddies move around a fluid very much as molecules move about in a gas, Prandtl derived an expression for momentum transfer in a fluid in which the mixing length ℓ plays a role roughly analogous to that of the mean free path in gas kinetic theory. This led to the relation:

$$\bar{\tau} = -\rho \ell^2 \left| \frac{d\bar{u}}{dy} \right| \frac{d\bar{u}}{dy} \qquad (19)$$

The mixing length ℓ is also a function of position; however, Prandtl achieved some success by letting ℓ be proportional to the distance from solid surface in the turbulent stream. A result similar to the above was obtained by G. I. Taylor's vorticity transport theory.

<u>von Karman's Similarity Hypothesis Expression</u> (G-7, D-10)

By means of dimensional analysis, von Karman deduced that the Reynold's stresses should have the form

$$\bar{\bar{\tau}} = -\rho K^2 \left| \frac{\left(\frac{d\bar{u}}{dy}\right)^3}{\left(\frac{d^2\bar{u}}{dy^2}\right)^2} \right| \frac{d\bar{u}}{dy} \qquad (20)$$

in which K is a universal constant whose value is given as 0.40 by some investigators and as 0.36 by others for tube flow velocity profile data.

<u>Deissler's Empirical Formula for the Buffer Region</u> (D-10,D-11)

Deissler proposed the following empirical expression for use in the neighborhood of the solid surfaces where the von Karman and the Prandtl equations are inadequate:

$$\bar{\bar{\tau}} = -\rho n^2\bar{u}y \left(1 - \exp\left\{- n^2\bar{u}y/2\right\}\right) \frac{d\bar{u}}{dy} \qquad (21)$$

in which n is a constant determined experimentally by Deissler as 0.124 from tube flow velocity distributions.

Use of the above theories can be summarized by indicating the range of flow over which they are applicable. Deissler in (D-10) and (D-11) reports excellent correlation and expands the expressions into workable equations. Also, other useful expressions have been developed considering the transport of solids in water. The range of application for equations (1), (20) and (21) best suited for each is as follows:

Equation (1) - Laminar sub-layer - Newton's Law.

Equation (21) - Buffer region (and laminar sub-layer) - Deissler's expression.

Equation (20) - Fully turbulent zone - von Karman expression

Friction Factor Correlations

In flow of fluids through pipes, the engineer is generally concerned with the relationship between pressure drop and the volume rate of flow. Also, in transport problems the relation between the velocity of the approaching fluid and the drag force becomes highly important. In many systems the velocity and pressure profiles cannot be calculated, so the engineer has sought through experimental data and correlations to relate pressure drop to quantity flow and drag force to velocity.

In considering flow through a pipe or around an object, the fluid will exert a drag force, F_D, on the solid surfaces. This drag force is related to the area of the system exposed and to the kinetic energy of the fluid stream, which may be written as follows:

$$F_D = A(KE)f \tag{22}$$

where f is a dimensionless proportionality factor known as the friction factor. For flow in a conduit, A is taken to be the wetted surface and KE is taken to be the quantity

$\frac{1}{2}\rho V^2$. From which for flow in pipes of radius R, f may be defined by:

$$F_D = (2\pi RL)\ (\tfrac{1}{2}\rho V^2)f \qquad (23)$$

Equation (23) can be written in terms of the pressure drop utilizing the cross-section area as:

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{L}{D} \cdot 4f \qquad (24)$$

This expression is a form of the Darcy-Weisbach equation which is fundamental to all hydraulics. Gravitational effects may be included by use of the expression

$$F_D = (\Delta p \pm \rho gL \sin \theta)A \qquad (25)$$

where θ is the angle of inclination to the horizontal.

By dimensional analysis and the use of the above and previous equations it can be shown that

$$f = \Psi\ Re \qquad (26)$$

This is a fundamental and important relationship which provides that for turbulent and laminar flow at any velocity in pipes of any diameter for fluids of any density and viscosity, data can be correlated in a plot of f versus $\frac{DV\rho}{\mu}$.

Expressions for the dividing point or transition range between laminar and turbulent flow have been developed and are

Laminar

$$f = \frac{16}{Re} \qquad Re < 2 \times 10^3 \text{ stable} \qquad (27)$$

$$Re > 2 \times 10^3 \text{ unstable}$$

Turbulent

$$f = \frac{0.0791}{Re^{\frac{1}{4}}} \qquad 2 \times 10^3 < Re < 10^5 \qquad (28)$$

Many other expressions have been developed for the particular problem under study at the time. For the present, however, the relationships shown are sufficient to permit further advance into the thesis subject.

Figure (4) shows a typical plot of friction factor versus Re. It should be emphasized that the general data discussed thus far has been for smooth surface circular pipes. If the circular pipes are rough then in the turbulent region, higher pressure drops are required for a given flow rate than would be indicated by the solid line of Figure (4). If k is the height of the protuberances, then the "relative roughness" would be expected to enter the correlation expressed in equation (26). Additional data has been plotted in Figure (4) for various values of k/D. Roughness tends to make f a constant over wide ranges of Re.

Diffusion (R-9)

An interchange of fluid between two neighboring zones necessarily involves the simultaneous interchange of every local

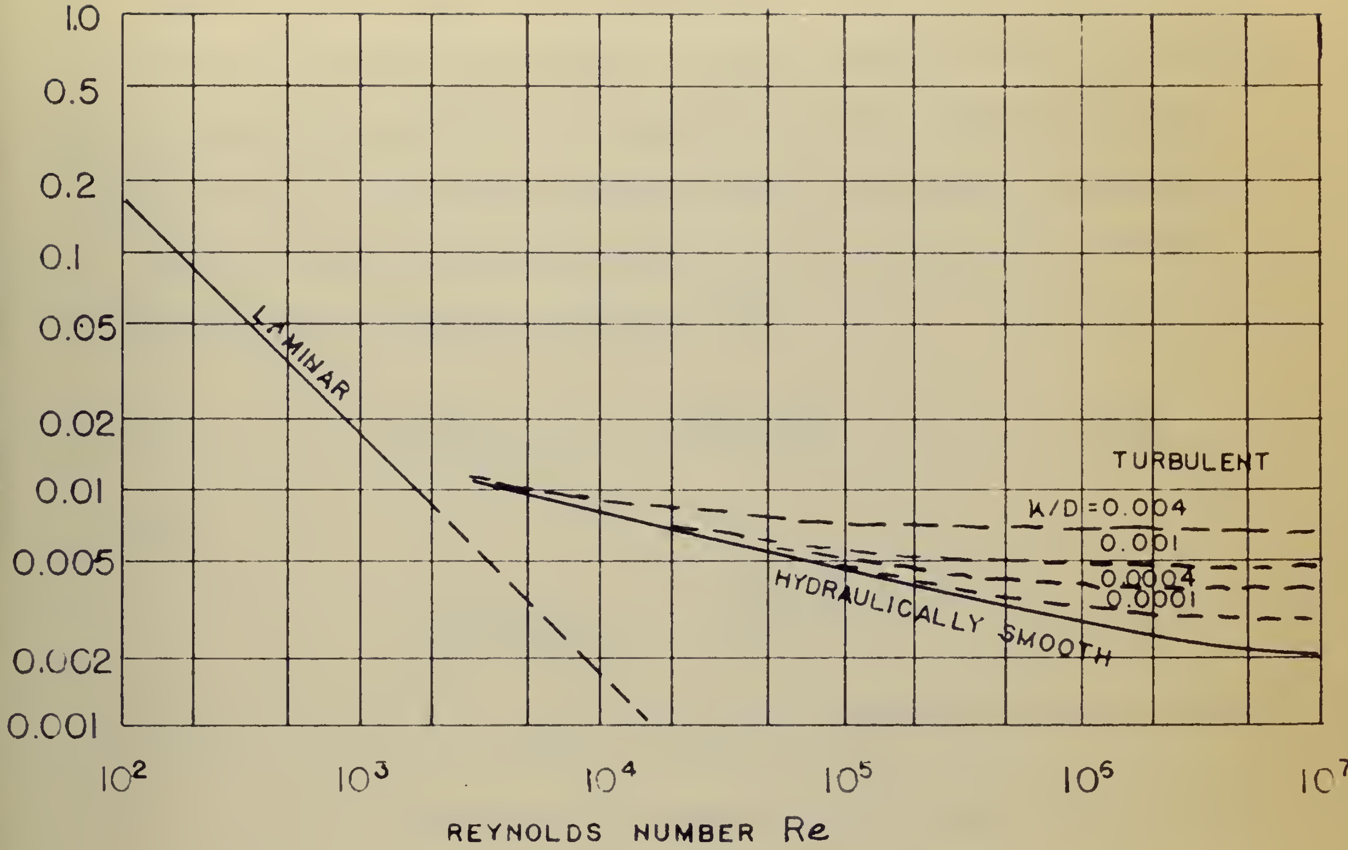

FIG. (4) - FRICTION FACTORS FOR TUBE FLOW (B-15)

characteristic which the fluid or flow may possess. The actual rate of increase or decrease evidently depends fundamentally upon two independent variables: the carrying capacity of the cross flow, and the extent of the difference of the characteristic in question between the two zones. This can be expressed mathematically in terms of mixing across a zone in which concentration of matter in either solution or suspension varies, as

$$M_{\text{laminar}} = -\beta \nu \frac{dC}{dy} \qquad (29)$$

or

$$M_{\text{turbulent}} = -\beta \varepsilon \frac{dC}{dy} \qquad (30)$$

where ε is $\frac{\eta}{\rho}$, β is a proportionality factor, and M is the rate of lateral transport. This relationship is of fundamental importance in recent investigations of sediment transport by water.

Transport of Solids by Water

It is generally considered that the turbulent effects of flow are the mechanism by which solids are transported in a water-solids mixture. In considering the problem, one finds that most investigators have worked with fairly large macroscopic particles which have a definitive settling velocity; and they have worked with particles of a single size rather than a truly wide range of particle size. In transporting

dredged solids, the particle size may range from minute colloidal particles to large rocks 27 inches or more in their major dimension. Analysis of the solids would seem to indicate that there must exist a minimum particle size below which all solids would remain in suspension for all practical purposes. This colloidal material, it would seem, would then contribute to physical, i.e., viscous property of the fluid. Of course, in any discussion of this problem, it is quite likely that the degree of solids concentration plays an important part in the viscous portion of turbulent flow as well as the turbulent eddy transport portion. At the other extreme of particle size, it is usually found that these larger particles are transported by turbulent transport in the leaps, skips and jumps of a saltation regime along the bottom of the channel or pipe.

It has also been found in most transport systems that a concentration gradient of solid material exists in vertical smooth pipe profile ranging from a maximum near the bottom to a minimum approaching the top. Of interest is that most horizontal smooth pipe profiles are reported as being a uniform concentration of solids.

In addition to the above, other limitations should be placed on the applicability of the solid-liquid transport or sediment transport theory, as it is often called. Actually, the items to be enumerated hereinunder are not necessarily

limitations as they may prove to be valid as a basis for operation of transport theory involving dredged solids. Assumptions, then, that most investigators state are:

a. Sediment water mixture is homogeneous.

b. The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

is applicable.

c. Sediment water mixture, in the limit case, is a homogeneous fluid of the Newtonian type.

d. Incompressible fluid.

e. Steady uniform flow.

Turning then to the actual development of the expressions for sediment transport, it is found that for turbulent flow equation (17) can be written as

$$\bar{\tau} = \rho u'v' \qquad (31)$$

if the viscous effects are neglected as minor which is perfectly valid. Then resorting to equation (19) and assuming that $u' \propto l \frac{du}{dy} \propto v'$ the following results:

$$\bar{\tau} = \rho l^2 \left(\frac{du}{dy}\right)^2 = \rho \left[l^2 \frac{du}{dy} \right] \frac{du}{dy} \qquad (32)$$

and

$$\bar{\tau} = \rho \varepsilon_m \frac{du}{dy} \qquad (33)$$

The term, $\rho \ell^2 \frac{du}{dy}$ is called the "eddy viscosity", while $\ell^2 \frac{du}{dy}$ is called the "momentum transfer coefficient", ε_m. O'Brien (O-1) first introduced an equation for the distribution of suspended sand in turbulent flow. By equating the rate of upward transfer, $-\varepsilon_s \frac{dC}{dy}$ of suspended sediment resulting from turbulent exchange and the rate of settling $\omega' C$ under gravitational force the following was obtained.

$$\varepsilon_s \frac{dC}{dy} + \omega' C = 0 \qquad (34)$$

From Ismail (I-1), the following general development of the transfer mechanism is set forth.

Von Karman (V-4) gave the analogy between the transfer of mass or heat and the transfer of momentum. He demonstrated that the coefficient of all these kinds of transfer should have the same form. Later Sherwood and Woertz (S-2) found that the transfer coefficient for water vapor in a turbulent gas stream was not equal to the momentum transfer coefficient ε_m, but that they seem to bear a constant relation to each other. Other investigators disclosed that this approximation can be assumed in heat transfer also. Therefor, it may be assumed that ε_s can be related linearly to ε_m:

$$\varepsilon_s = \beta \varepsilon_m \qquad (35)$$

in which β is the coefficient of proportionality.

For two dimensional flow

$$\tau = \tau_o \left(1 - \frac{y}{y_m} \right) \tag{36}$$

in which τ_o is the shearing stress at the boundary and y_m is the vertical distance to the maximum velocity, i.e., the plan of zero shear. Thus, equation (36) can be combined with equation (33) to give:

$$\varepsilon_m = \frac{\tau_o \left(1 - \frac{y}{y_m}\right)}{\rho \frac{du}{dy}} \tag{37}$$

The von Karman universal velocity defect law (B-3) is:

$$\frac{u - u_{max}}{u_f} = \frac{1}{K} \log_e \frac{y}{y_m} \tag{38}$$

in which $u_f = \sqrt{\frac{\tau_o}{\rho}}$. Therefore, when $\frac{du}{dy}$ is evaluated from equation (38) and applied to (37), the result is:

$$\varepsilon_m = K u_f y \left(1 - \frac{y}{y_m} \right) \tag{39}$$

and from (35)

$$\varepsilon_s = \beta K u_f y \left(1 - \frac{y}{y_m} \right) \tag{40}$$

Equations (39) and (40), according to Ismail (I-1) give zero values for both ε_m and ε_s at the center of the channel.

Many investigators maintain that the transfer coefficients are not zero at the center, and therefore, the equations are not valid at this point. Von Karman (V-3) explained how these relations were derived from the assumption of similar flow pattern; then he stated that in the center part of a pipe the similarity assumption cannot be correct. Brooks and Berggren (B-4) showed how Prandtl tried to correct that region and how the experiments of Nikuradse gave definite values of ε_m at the center of the pipes in his experiments. They showed that two reasonable assumptions which can be made at the center are either to take ε_m as a constant according to Sherwood and Woertz (S-2) or to assume an error curve for ε_m. If ε_s is constant over a certain region, equation (34) can be integrated over that region to give

$$\log_e \frac{C}{C_a} = \frac{w}{\varepsilon_s} (y - a) \qquad (41)$$

in which C_a is the concentration at any arbitrary reference level $y = a$.

If ε_s follows equation (40) in another region, equation (34) will integrate to give:

$$\log_e \frac{C}{C_a} = \frac{W}{\beta K u_f} \log_e \left[\left(\frac{y_m - y}{y} \right) \left(\frac{a}{y_m - a} \right) \right] \qquad (42)$$

or

$$\frac{C}{C_a} = \left[\left(\frac{y_m - y}{y} \right) \quad \left(\frac{a}{y_m - a} \right) \right]^Z \qquad (43)$$

in which $Z = \dfrac{w}{\beta K u_f}$

Vanoni (V-1) verified equation (42) and showed how well it fitted the experimental points in an open channel. He also showed that the existence of suspended load tends to suppress or damp out the turbulence causing a decrease in the value of K, the von Karman universal constant. Of importance is that ϵ_s depends on the size of sediment in suspension. Fine sediment tends to make ϵ_s greater than ϵ_m; coarse sediment has an opposite tendency.

As has been previously stated, the value of K is generally been reported as ranging from 0.36 to 0.40. However, Ismail (I-1) reports for sediment laden flow that K decreased to a low of 0.20 for a concentration of sand equal to 43 grams per liter of solution. The decrease of K reportedly is attributed to damping of the turbulence. Ismail further reports that β = 1.5 for 0.10 mm sand and 1.3 for 0.16 mm sand.

In summary, reported results indicate that equation (34), the basic equation for sediment distribution and plot of the distribution curve of ϵ_s in a vertical profile of the channel can accurately define the concentration distribution of the

$$[illegible]$$

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flow system. Practically, the difficulty in this procedure is that K and β cannot yet be predicted from theory and they must either be assumed or determined experimentally. In spite of this, the developed theory of the sediment (solids) transfer mechanism represents a definite advance in thinking and procedure which should, with further practical research, permit rapid evaluation of sediment transport.

Dimensional Analysis for Sediment Transport

It has already been noted that most of fluid dynamic formulae and relationships presented are empirical in nature. Exact rigorous solution of the problem of turbulent flow cannot at present be made. This statement is even more to the point for problems involving liquid-solids transport phenomena. Empirical relationships are generally established from experimental results. On the other hand, there are correlations which have been established from purely deductive analyses. It can be definitely stated that empirical and deductive relationships are seldom reconciled without a compromise.

Dimensional analysis has been used by many investigators to provide equations which could then be examined in the light of actual physical behavior of the flow system and with real experimental data. An analysis of the problem of sediment transport through pipes is given from the results of Garde (G-1).

The variables entering the problem can be classified

into four categories.

(A) Sediment

ρ_s - Density of sediment

d - Mean diameter of sediment

σ_g - Geometric standard deviation

S_f - Shape factor

(B) Fluid

ρ_w - Density of fluid

μ - Coefficient of viscosity

(C) Flow

V - Mean velocity of flow

J - Hydraulic gradient

C_T - Average sediment concentration

(D) Boundary

D - Diameter of pipe

S - Slope of pipe

ϕ - Variable characterizing form

Of these variables only J and C_T are dependent. Their functional relationship can be written as

$$\Psi_1\,(\rho_s,\ d,\ \sigma_g,\ S_f;\ \rho_w,\ \mu\ ,\ V,\ C_T,\ J,\ D,\ S,\ \phi) = 0 \qquad (44)$$

Garde drops σ_g and S_f as not significant for his experiments. Then, selecting ρ_w, V and D as repeating variables, and

applying the π theorem gives

$$J = \Psi\left(\frac{\rho_s}{\rho_w}, \frac{d}{D}, \frac{VD/\mu}{\rho_w}, \frac{V^2}{gD}, C_T, \frac{\phi}{D}\right) \tag{45}$$

Also, since for all the data used in Garde's work, sediment with specific gravity of 2.65 was used and the continuous medium was water $\frac{\rho_s}{\rho_w}$ was constant. Hence,

$$f = \frac{J}{\frac{V^2}{2gD}} = \Psi\left(\frac{d}{D}, Re, C_T, \frac{\phi}{D}\right) \tag{46}$$

In the phenomenon of surface resistance in pipe flow, it has been found that a dimensionless parameter $Re\sqrt{f}$ is very significant. Since the relative thickness of the laminar sub-layer in a pipe (i.e., the ratio of the thickness of laminar sub-layer to pipe radius) is inversely proportioned to $Re\sqrt{f}$, according to Garde (G-1) it is logical to assume that $Re\sqrt{f}$ may also play a significant role in sediment transport through pipes. The functional relationship for $Re\sqrt{f}$ may be expressed as

$$Re\sqrt{f} = \Psi\left(\frac{d}{D}, C_T, \frac{\phi}{D}\right) \tag{47}$$

If E (W-6) is defined as energy required for transporting a unit weight of sediment per second per foot of pipe, then a functional relationship between the different variables

will be of the form of

$$E = \Psi'\left(W, \frac{\phi}{D}, Re, \frac{d}{D}\right) \qquad (48)$$

where W is the rate of sediment transport in pounds per second. Note should be taken of the fact that the expression above is a dimensional equation since it contains a dimensional parameter W, the sediment discharge in weight per second.

Thus, dimensional analysis has shown that, in studying the problem of sediment transport through pipes, the significant parameters will be: J, $\frac{\rho_s}{\rho_w}$, $\frac{d}{D}$, C_T, $\frac{V^2}{gD}$, Re, $\sqrt{f}$, Re$\sqrt{f}$ together with the parameter describing the boundary form. These parameters together with their combinations should be adequate to solve problems, such as influence of total sediment load on the resistance coefficient.

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CHAPTER III. REFERENCE STUDY

A reference study has been made of the subject matter of this thesis. In this study much use has been made of condensed versions of reference material written by other authors where verified by the present author. No attempt has been made to list the references studied in any kind of subject order, rather a chronological sequence has been adhered to.

It should be pointed out that the reference study by no means represents all of the material available for reference. An examination of the bibliography, which is a part of this thesis, provides an even greater reference listing.

Maltby (M-4) 1905, reported on the results of friction loss investigations on the discharge lines of eight dredges of the Mississippi River Commission. The dredges operated on the portion of the river below the junction of the Mississippi with the Ohio, at Cairo, Illinois. Tests were made from 1896 through 1905. With the sediment all in suspension, apparently in smooth pipes about 24 inches in diameter, the friction factor, f, was found to be about 0.015. Specific conclusions were not presented.

Blatch (B-7) 1906, appears to have been one of the first to give the problem of sediment transport in pipes the

consideration it deserves. Tests were made to determine the head loss along a pipe 27 feet long with a 1 inch nominal diameter. A 1.0546 inch diameter brass pipe, transporting sands of about 0.60mm and 0.20mm median sieve diameter, and specific gravity of 2.64 was studied. Also examined was a 1.0420 inch diameter galvanized iron pipe carrying 0.60mm sand. It was demonstrated that, for a given total sediment load, head loss deviated further and further from the clear water head loss curve as the mean velocity decreased. Defining the "economical velocity" as that at which the head loss due to a given mixture of sand is a minimum, it was concluded that the economical velocity for a 1 inch pipe was 3.5fps. Furthermore, it was observed that there existed a transition in the regime between 3.5 and 4.0fps. The transition increased in length with an increase in grading of the material. For fine and uniform sediment, the transition zone is narrow.

Orrck and Morrison (O-4) as early as 1921 made estimates on the economic practicability of pumping anthracite and bituminous coal as far as 320 miles. Excluding water costs, all the estimates were equal to or less than 3 mills per ton-mile for pumping 6 million tons per year.

Clifford (C-12) 1924, attempted to analyze the flow of sewage sludge by an analogy with the laws of viscous fluid

flow. A kinematic viscosity for the sludge was determined by comparing the flow of the sludge containing 90% moisture to the flow of glycerine. This comparison was made with a 5/16 inch diameter tube, 5.25 inches long. Applying the kinematic viscosity determined as given above, theoretical friction losses were computed for an 8 inch line, at Calumet, Illinois, pumping sludges containing 90% water. Calculations checked with the measured friction losses.

Cramp (C-11) 1925, set up a design criteria for pneumatic conveyors. Plants were classified as pressure, suction, or a combination. The factors to be considered were the pressure differential, the sediment-pipe friction, the air-pipe friction, the force to support the material, the force to support and accelerate the air, and the force to accelerate the material. A very lengthy equation involving all the above factors is given for the case of vertical pipes. It is stated that a modification of the equation would be applicable to horizontal lines. Measurements and circulations differed less than 4%. Horsepower was computed by using isothermal compression.

Gregory (G-2) in 1927 determined the head loss when pumping a slurry through a horizontal 4 inch pipe. Losses due to fittings were also evaluated. The test line was about 250 feet long. The sediment was made up of 70% (a little sand)

clay, 24% carbonate of lime, and 6% hydrate of magnesia and hydrates of iron oxide. The material ranged from colloidal to microscopic in size and did not settle quickly nor cake in the pipe. Plotting J versus V, it was found that above a certain velocity the plot agreed with that of clear water, with total loads ranging from 5 to 29% by volume. (18.6 to 35.3% by weight.) For lower velocities, of a particular total load, the head loss remained constant. The maximum velocity for which J was independent of V, for a constant load, was named the "critical velocity". It was also the most economical velocity in terms of power consumption.

Mikumo, Nishikara and Takahara (M-6) wrote a paper in 1933 on their tests of pipeline head loss with copper ore slimes pumped through a $1\frac{1}{2}$ inch pipeline. The ore had an average specific gravity of 3.5. The median size of the material was 0.0042 inch. The equipment consisted of a recirculation system powered by a centrifugal pump. The length of the horizontal test section for measuring pressures was 1 meter. Methods for the measurement of velocity and concentration are not adequately explained.

Brautlecht and Sethi (B-9) 1933, presented data on head loss in a horizontal 1 inch pipeline transporting unbleached sulphite pulp. The pulp was screened through a No. 10 flat screen. Concentrations were 0 to 1.57% by bone dry weight.

The pipe plugged at a concentration of 3%. Velocities ranged from 2.0 to 8.8 fps.

<u>Thoenen</u> (T-2) discussed in 1936 the advantages and disadvantages of pipeline transportation of sand and gravel. An example of typical pumping performance curves and a list of the characteristics of mixtures are given. Included also are friction losses for fittings and some dredge pipeline data.

<u>Traxler</u> (T-4) 1937, pointed out that the flow properties of dilute suspensions of clay and other minerals depends to a large extent on particle size, size distribution and shape. It was concluded that there is a simple relationship between the concentration of solids and the viscosity of the suspensions.

<u>O'Brien</u> and <u>Folsom</u> (O-2) 1937, published one of the classic papers on the subject of pipeline sediment transport. They ran a series of tests with 2 inch and 3 inch standard black wrought iron pipe. The pipelines were horizontal. Three sizes of sand were used, ranging from about 0.0065 inch to 0.050 inch median size. A sand and water mixture was circulated, and concentration, hydraulic gradient and mean velocity were measured. In analyzing this data it was found that the Darcy-Weisbach equation $J = fV^2/2gD$, where g is the gravitational field, was valid for non-homogeneous mixtures as well as for homogeneous fluids. Two minimum velocities were defined: (1) a "critical velocity", at which

the head loss begins to differ appreciably from the head loss for clear water in the same pipe, and (2) the velocity of incipient clogging. It was concluded that the critical velocity was a velocity below which a part of the material was transported by rolling along the pipe and as the velocity is decreased, more and more area of the pipe becomes obstructed by the rolling sand until conditions of incipient clogging occur. Furthermore, the most efficient velocity for pumping a given total load is the minimum velocity that will move the material through the pipeline, which was independent of sand size for the range reported. In a discussion of the effects of solids on the turbulent flow, they believed the concentration of the material might not be the same across horizontal surfaces because the particles tend to fall to the lowest point, including a double spiral secondary flow, downward at the center and upward along both sides.

Wilhelm and others (W-1) 1939, published friction loss data on the pumping of cement rock and filter-gel suspensions. Horizontal pipes 0.75 inch, 1.5 inch and 3 inches in diameter, 27 feet long, were tested with water sediment complex. A reasonable correlation of all these data was obtained on an f-Re diagram by substituting an apparent viscosity, determined with a rotating viscosimeter, for the fluid viscosity.

Howard (H-6) 1939, studied the transportation of sediment through 2 inch and 4 inch pipes. The length of pipes under test

was about 14 feet. The following sediments were tested:

Commercial Name	50% Size mm	Classifications
Pea Gravel	2.50	Medium Gravel
Pearl River Sand	0.40	Medium Sand
Laboratory Loess	0.024	Silt
Buck Shot	0.001	Clay

The quantities measured were pressure loss, sediment concentration, and velocity. Important conclusions drawn from these tests are as follows:

a. There are three distinct ways in which sediment is transported through pipes: by rolling when the velocity is small; by jerking when the velocity is medium; and by the motion of all the particles over the entire cross-section of the pipe when the velocity is large.

b. For pipes carrying sand, f decreases with an increase in velocity.

c. Values of f will increase with an increase in solids concentration for any given velocity.

d. Economic velocity for transporting solids depends upon the character of sediment to be carried, and each class of sediment will probably have a different economic velocity for the same size of pipe.

e. A very fine sediment should be transported with much less head loss than large sediment.

f. Extension of results from a small pipeline to a pipeline of greater diameter must be qualitative and not governed by any law of corresponding velocities.

Durepaire (D-7) 1939, in the discussions of Howard's paper (H-7) described the results of tests carried out at Nantes Harbour. The inside diameter of the pipe was 2.05 inches and Loire River sand with a maximum grain size of 0.30mm was used. The results can be summarized as follows:

a. With all the sediment load in suspension and within the range of concentration encountered in the experiment (up to 40% by volume), the head loss expressed in feet of mixture was the same as that for clear water, irrespective of concentration, except at the state when deposition was impending.

b. Cirtical velocity (i.e., velocity at which deposition begins for a given concentration) and economic velocity (i.e., velocity at which head loss is minimum for a given concentration) occur approximately at the same velocity.

c. No jerking motion of the sediment was observed.

d. It was found in the partial deposition phase that for a constant concentration of sand, head loss is greater when total discharge decreases. At the same time the height of deposited sediment increases. The deposit of sediment in a given run was of rather uniform depth because there was no jerking motion.

Durepaire (D-16) 1939, in a contribution to the study of sediment pumps, carried out a complete and clear analysis of stability in sediment-water mixture transport lines. Hydraulic gradient was plotted versus discharge of mixture for various arbitrary total loads, giving a typical curve in which there always exists a minimum head loss for each total sediment load. A hypothetical pump characteristic curve was superimposed on the J-V diagram. The stability of the plant operation, which decrease the probability of pipeline clogging, was presented for any pump having a Q equal a constant characteristic, and conversely.

Wood and Bailey (W-7) 1939, presented the research done on transportation of sand and linseed sediments by air in horizontal pipe of 2.9 inch diameter and 25 feet long. The data was taken in the range where the sediment moved in saltation and it was found that head loss is a linear function of C_T.

Caldwell and Babbitt (B-6) wrote a paper in 1939 on the laminar flow of sludges. Experiments were with 1 inch, 2 inch, and 3 inch horizontal pipes carrying sewage sludge. It was concluded that clay slurries and sewage sludges behave as true plastics; and therefore, a yield stress and a coefficient of rigidity were necessary to describe the "fluid", in place of viscosity used for a Newtonian fluid. The yield stresses and the coefficient of rigidity were independent of the diameter and roughness of the pipe in which they were measured, but did depend

on the concentration of suspended material, size and character of sediment particles, nature of the continuous phase, temperature, thixotropy, slippage, aggitation and gas content of the sludge.

Chatley (C-8) 1940, discussed the elements of power consumption in the transport of granular solids in suspension. The significant ones are lifting the solid particles, accelerating the grains, sustaining the grains, friction against the pipe walls, accelerating the fluid, fluid friction, and losses at the bends and valves. Sample calculations were presented for a 6 inch pipe, 50 feet long, transporting 48 tons of grain per hour. Air was used as the fluid. The computations indicated that roughly three times as much horsepower is required for horizontal transport of grains as for vertical conveyance. The major portion of the difference in power seemed to be due to the higher power requirement in the horizontal pipe needed to accelerate the grains. It was thought that slope is a significant variable in pneumatic transport when it deviates from the vertical

Howard (H-7) presented, in 1941, one of the few papers on large artificial roughnesses placed in a pipe for the purpose of improving sediment carrying capacity. In order to determine optimum rifling, loss tests were run on a 4 inch 20 gage steel pipeline, carrying water and 0.39mm sand from the Pearl River. It was determined that the length of rifling should be 1/3 the pipe length. One of the best riflings consisted of the above

length, with three riflings spaced at 120 degrees around the pipe and having a pitch of 10 diameters. The roughness height was D/8, where D is the pipe diameter. A similar rifling was installed in a two inch pipe and tests made with the same sand. It was hoped in this manner that a design could be made for pipes of 30 to 32 inches. Using the optimum rifling, head loss tests were made using 0.023mm silt and 2.48mm pea gravel. The conclusion was reached that rifling will increase the efficiency of plants pumping coarse sand and gravel, for the range of velocities customarily used, but will decrease it for silt and clay; i.e., efficiency will be increased for any material large enough to settle and travel along the bottom of the pipe. If the velocity becomes sufficiently high when transporting a given material, the energy loss due to the rifling may overshadow the decrease of energy consumption due to the induced suspended flow, and a decrease in efficiency will result.

Danel (D-5) in a discussion of Howard, pointed out that a density gradient in a pipeline could cause a damping of turbulence, just as density variations with height causes the calmness of the atmosphere at sunset. He also mentioned the importance of the size of the material in this phenomenon. If the particles are coarse, their continued falling through the fluid creates the turbulence and there is little chance that density variation can reduce turbulence to the point where the head loss is less than that for water at the same velocity.

Caldwell and Babbitt (B-6) reported in 1941 on the extension of the laminar flow tests of 1939, with sewage sludge, to the turbulent regime. New standard black steel pipe 0.5 inch, 1 inch, 2 inch, and 3 inches in diameter was employed. Except for the 0.5 inch pipe, the test reach was 21 feet long preceded by at least 40 pipe diameters for establishing the flow pattern. The friction loss was measured for several velocities with each of the eight sludges tested. In analyzing this data it was found that a diagram of f versus $Re = VD\rho/\mu$ was adequate to solve friction loss problems when the velocity was greater than a certain magnitude. For velocities less than this, the hydraulic gradient deviated appreciably from that for clear water in the same pipe. Another important conclusion was that, for sludges composed of water and suspended material, the viscosity of the dispersion is nearly the same as that of water, so that the common hydraulic formulas can be used in evaluating turbulent flow friction losses when pumping sludges.

Wilson (W-5) proposed an elementary theory in 1942 of pipeline transport of non-colloidal inert solids, assuming that the particle settling velocity relative to the transporting fluid is known. A relationship was written between the work done by the liquid on the particles and the decrease in potential energy of a settling particle. Using the Darcy-Weisbach equation and assuming the energy gradient may be divided into two parts, the result of the energy calculation was the equation:

$$J = \frac{fV^2}{2gD} + \frac{\gamma_s - \gamma_w}{\gamma_w} \quad (1 + A_1) \quad \frac{C_T \omega}{V}$$

for the hydraulic gradient when sediment was being transported, where γ_s and γ_w are the unit weight of the sediment and water respectively. The fall velocity of the sediment particles is ω and A_1 is a constant. By assuming C_T, ω, f, A_1, and D constant, a condition for deposition was derived. The condition is $\omega/\sqrt{JgD/4} = 1$ for horizontal pipes. That actual cases would differ from this, due to non-uniform values of shear over the cross-section, was recognized.

Wilson (W-6) 1945, analyzed the data taken by Blatch, Howard, and others. This produced the dimensionless parameter to describe the energy required for sediment transport:

$$E = \frac{J}{\frac{C_T}{100}}$$ = Energy required per foot of pipe per pound of sediment transported.

His analysis of the data led to the following general conclusions:

a. Since E is proportional to J/C_T, the most efficient transportation of sediment through pipes, as far as energy required is concerned, is when C_T is a maximum, for a given amount of sediment to be transported.

b. A limit to the above statement is the point beyond which one cannot decrease the discharge for given solids

of a particular size without clogging the pipes.

c. Analysis of Howard's data (H-7) for pipe with rifling showed that with a given quantity of mixture flowing, more energy per unit mass of the solids transported is required to transport a small solids load than a large solids load.

Vanoni (V-1) 1946, published results of the experiments conducted by him on transportation of suspended sediment by water in a Flume 33.25 inches wide and 60 feet long, the slope of which could be adjusted. The experimental sediment distribution in the vertical was compared with the theoretical distribution. The formula

$$\frac{C}{C_a} = \left[\left(\frac{D_1 - y}{y} \right) \left(\frac{a}{D_1 - a} \right) \right]^{Z_1}$$

was obtained on the assumption that ϵ_s and ϵ_m are equal. His conclusions are summarized as follows:

a. The distribution of relative concentration of suspended load is given by the above equation, but the value of Z_1 given by the theory does not agree with the value of Z_1 that fits experimental data.

b. The above disagreement is attributed to the action of random turbulent fluctuations in suspended sediment and "slip" between the fluid and the sediment as the sediment is

accelerated. This makes ε_s differ from ε_m.

c. For fine material, ε_s tends to exceed ε_m; for coarser material the opposite tendency is found.

d. Suspended load decreases the value of K, which characterizes the effectiveness of turbulence in transferring momentum. Reduction of K means the mixing is less effective and would indicate that the sediment tends to suppress or drop out the turbulence.

Vogt and White (V-2) 1948, published a paper on "Friction in the Flow of Suspensions-Granular Solids in Gases Through Pipes". Friction losses were studied in 0.50 inch pipe carrying sand sizes 0.0088 inch, 0.0138 inch, 0.0018 inch and 0.0287 inch; steel shot 0.0165 inch; clover seeds 0.046 inch; and wheat 0.158 inch in diameter. Both horizontal and vertical pipes were used. Using $(J - J_o)/J_e$ as a significant parameter it was found that

$$\frac{J - J_e}{J_e} = A_1 \left(\frac{D}{d}\right)^2 \left[\frac{\rho_a}{\rho_s} \cdot \frac{q}{Re}\right]^{A_2}$$

where A_1 and A_2 are constants and q is the ratio of solids to air.

Danel (D-12) presenting some theoretical considerations in 1948, again put forth the concept of "evening calm" with respect to a flow having a marked density gradient. It was concluded that the amount of energy dissipated in maintaining the sediment in suspension was about equal to the decrease in energy

dissipation due to the damping of the turbulence, i.e., the total head loss for a mixture is approximately the same as that of a fluid of the same average density. The concept of a plastic film near the wall was postulated for the transport of very fine materials. This is the usual laminar sub-layer, into which the sediment has diffused until the layer is plastic in character.

Hariu and Molstad (H-5) in 1949 described the experiments in vertical glass tubes of 0.267 inch and 0.532 inch diameters. The object of investigation was to study the effect of gas velocity and concentration on pressure drop. The sediment sizes used were Ottawa sand 0.00165 feet and 0.00117 feet; sea sand 0.00090 feet and 0.0070 feet; micro-spherical cracking catalyst 0.00036 feet; and ground cracking catalyst 0.00036 feet. The pressure drop was divided into that due to a gas alone when no sediment is present, that due to friction loss from contact between sediment and pipe, and that due to solids-static loss. The solids-static loss was described as the pressure drop in the gas due to supporting dispersed solids in the given length of vertical tube.

Farbar (F-1) 1949, in studying the flow characteristics of the solids-gas mixtures in horizontal and vertical pipes, used a 17mm tube with material ranging from 8 microns to 220 microns in diameter. It was found that at large concentrations there was a tendency for the pressure drop to be independent of

concentration. He also found that the flow characteristics of solids-gas mixtures in which size distribution covers wide ranges, differ considerably from mixtures of narrow size range.

Durand (D-13) reported in 1951 on the hydraulic transport of gravel and pebbles. The material was sieved into categories between the following limits: 2.3mm - 5.25mm - 9.9mm - 15.5mm - 20mm - 25mm. A 104mm horizontal pipe was used for the tests. The head loss varied with total load for all the gravels studied, but seemed independent of the mean diameter of the grains. Furthermore, for a given velocity in the range usually found in hydraulic conveying work, the head loss was higher than that for clear water. A classification of "sand" sediments was proposed, based on the characteristic plot of head loss versus velocity plot for each class. The classes are: (1) silts and fine sands which follow Stokes Law, (2) coarse sands, and (3) gravels and pebbles that follow the Rittinger Law. This classification emphasizes the significance of the parameter w/V.

Durand (D-14) discussed in 1951 some experimental work on the pumping of fine ashes from a power plant. The material covered a wide size range, 2 to 100 microns, with a specific gravity of 2.5. The pipeline was 250mm in diameter. Total load was as high as 300 grams per litre. It was felt that due to the heterogeneity of the sediment, no precise conclusions

could be made until additional data on screened sediments were made available.

Collins (C-14) in 1951 provided a paper which gives a practical, though empirical approach to the operation of hydraulic dredge pipelines. His paper included the report of a study of several months duration on the dredge Marshall C. Harris in 1934 and 1941. The results of the investigation are reported in the form of a summary problem. Complete data or physical operating characteristics are not presented. Collins states that the following rules are applicable to hydraulic dredging.

1. The pump capacity (gpm) passing through the pump is directly proportional to the speed (rpm).

2. The total pressure (psi) developed by the pump varies as the square of the rpm of the runner.

3. The horsepower (bhp) required will vary as the cube of the speed.

4. The pressure developed by the pump runner will increase in proportion to the specific weight of the fluid.

5. In a pipeline the pressure head loss due to turbulence and friction varies as the square of the velocity of the fluid.

6. In a horizontal pipeline and when the velocity is sufficient to keep the solid particles in homogeneous suspension, the friction loss varies directly as the specific weight of the fluid.

Collins also states that the nature of pipeline flow can be considered to have three distinct stages. The first when the resistance to flow is caused by clear water only. The second stage occurs when the solids are homogeneously suspended. The third stage must be considered a phenomenon, and when it occurs, the line resistance can increase 100% over that of the first stage in a few seconds.

Collins also points out that the absolute accuracy of Rules 1 to 6 might be questioned by various hydraulic experts and it may even be well to concede a possible deviation of from 5 to 10% in the anticipated ratings of the pump.

Contained in the analysis presented by Collins is a complete outline for computing dredge pipeline capacity, velocity, pump characteristics, etc.

Soleil and Ballade (S-11) carried out tests in 1951, reported in 1952, on the Nantes dredging operations. Head loss, rate of material transport and depth of deposit in 580mm and 700mm pipes were measured. The test section was about 100m long, horizontal and located at the end of the main dredge discharge line. Conclusions were that the pipe diameter ought to be chosen as large as possible in order that the width of deposit represents a considerable part of the pipe diameter. The larger pipe had the lesser head loss and carried more sediment per cubic foot of discharge than a smaller pipe with equal mixture discharge. The head loss was very sensitive

to sediment size. Material over 0.3mm affected the head loss, but material less than 0.3mm diameter did not influence it.

Tison (T-5) wrote in 1952 on tests with a chemical plant residue of about 0.05mm which was pumped through 1 inch and 2 inch diameter pipes. Concentrations by volume of 15 and 30% were used. Head loss was measured in feet of water because the specific gravity was always less than about 1.15. The sediment reduced the resistance coefficient f considerably, at least for Re numbers greater than 10,000.

Ward (W-10) in 1952 published his doctoral thesis on the flow of air-water-clay and water-clay mixtures through 3/4 inch, 1 inch and 1½ inch nominal size copper pipes. The solid material used was Kaolin clay with specific gravity ranging from 2.45 to 2.51 at 25°C. An average value of 2.48 was used. Average particle size was approximately 5 microns.

The pressure drop increased at a fixed flow rate linearly with increasing amounts of solids up to the higher concentration. It then increased rapidly with small increases in solids concentration. This indicates that at the higher concentrations the flow properties of the suspensions are changing rapidly. Sample concentrations were produced which behaved as pseudo-plastics and Bingham plastics.

Ward also concluded, after examination of the "turbulent viscosity" that the usual Newtonian-friction factor Reynolds number relationship is approximately true for water-clay suspensions

and the turbulent viscosity computed from this relationship has no valid significance.

Craven (C-2) 1952, presented the results of studies at the University of Iowa with 60 feet lengths of 5.5 inch ID and nominal 2 inch diameter plastic tubes. Tests were run with three grades of uniform quartz sands (9.25mm, 0.58mm, and 1.62mm). The slope of the tube could be varied at will. By dimensional considerations, it was demonstrated that the bedload transport could be adequately described by two dimensional relations. The experiments were designed to determine these functional relationships. The investigations were not sufficiently extensive to attain the original objective, but certain interesting observations were made. It was found that the bed configuration, for each of the sands, evolved through the same pattern but the value of C_T at which a given change occurred was different for each grain size. It seemed that the higher the value of V/ω', the greater was the tendency for sediment to be lifted into suspension and the less the tendency for it to travel by dunes. If the actual shear values are far greater than critical in the open channel flow, the majority of the bed load equations take the form

$$J \propto \frac{\Delta \gamma}{\gamma_w} (C_T)^n$$

where n varies from 1/2 to 2/3. It was found that a similar relation holds good for pipe flow, the equation taking the form

$$J \approx 0.606 \frac{\Delta\gamma}{\gamma_w} (C_T)^{2/3}$$

Ambrose (A-2) 1952, extending the research of Craven (12) with the same basic equipment, investigated the case of free surface flow in pipes. Two relationships to define the phenomenon of sediment transport were derived by dimensional analysis: (a) a transport function relating the discharge to the geometry and to the other characteristics of flow, channel, and sediment, and (b) a discharge function relating the discharge to the resistance to flow of the sand bed and the pipe wall. Analysis of this data indicated that the transport function appears to be dependent solely upon the main geometry. No discernable effect due to d/D was evident. It was found that the transport function reached a maximum value of approximately

$$2.9 \left(\gamma_s / \gamma_w - 1 \right)^{2/5}$$

Deposition would not occur for values of the total load greater than this. The parameter γ_s / γ_w was constant. The discharge function seemed adequately defined if k/D and Re were neglected (k is an equivalent uniform sand roughness).

a. Transport function

$$\frac{\dfrac{Q}{D^2}}{\left(\dfrac{\gamma_s}{\gamma_w} - 1\right)^{\frac{2}{5}} Q_s^{\frac{1}{5}} g^{\frac{2}{5}}}$$

which is dependent on Re, γ_s, y/D, y_s/D, d/D and K/D where y and y_s are the depths of flow and sand bed respectively. Q_s is the absolute volume rate of transport.

b. The discharge function

$$\frac{Q}{g^{\frac{1}{2}} s^{\frac{1}{2}} D^{\frac{5}{2}}}$$

depending on the same parameters described above.

Head (H-11) 1952, working in the paper engineering field with non-Newtonian fluids of pulp and sulfite, introduced terms to replace the dynamic viscosity which is strictly applicable only for Newtonian fluids. The shear diagram was a straight line with a non-zero intercept for zero shear over the range investigated. This led to defining the physically meaningful "slope viscosity" as the slope of the shear diagram. An "apparent yield stress" was defined as the intercept on the shear diagram when the shear was zero. A "shear criterion" was introduced for non-Newtonian suspensions.

Kestlicher (K-2) 1952, studied head loss in a 101.6mm cast-iron pipe 61m long transporting very fine sediment. Specific gravity of the mixture ranged from 1.00 to 1.28. As in earlier investigations, it was found that the head loss curve corresponded to that of clear water for velocities greater than a value depending on the total head. In this case the asymptotic curve was determined after the tests had been completed, and a very smooth coat of sediment covered the pipe roughnesses. This coating reduced the resistance about to that of hydrodynamically smooth pipe.

Dougherty (D-6) made a survey, reported in 1952, on the problems presented by the pipe transportation of coal by water. After reviewing the opinions of manufacturers of centrifugal sand pumps, and some economic studies available in the literature or files of manufacturers, the conclusion was reached that coal could be transported economically in pipelines, especially in large tonnages. Further conclusions were that, before coal pipelines could be built, much more pumping performanc data is needed, pumps need more development, data on errosion of pumps is lacking, and data on the most economical linear pumping velocities and pressure drop must be determined for various coal mixtures.

Ismail (I-1) published a paper in 1952 on the studies conducted in a rectangular pipe 10.5 inches wide and 3.0 inches

deep. The conduit was 40 feet long. Sediment was recirculated. Sands having sedimentation diameters of 0.16mm and 0.10mm were used. Measurements were made of the discharge, velocity profile, concentration profile and head loss along the conduit. The analysis of data was based on considerations of the effect of the sediment on f, ε_s, ε_m, and K. Conclusions were:

a. The von Karman constant K decreases when the total load in suspension is increased. Both of the sands used in the experiments have nearly the same effect on K, which decreased to as low as 0.20 when C_T was 43 grams per liter.

b. The change in K does not follow the change in concentration from point to point over one cross-section, but it varies from section to section maintaining a constant value over each section.

c. The value of ε_m is affected by the presence of sediment only through changes in K.

d. The Darcy-Weisbach resistance coefficient, f, is not affected by the pressure of sediment up to a point where sediment load is great enough to form dunes; then f becomes greater than that for clear water.

e. The sediment transfer coefficient ε_s is approximately equal to 1.5 ε_m for the 0.10mm sand and 1.3 ε_m for the 0.16mm sand.

An independent analysis by Laursen and Lin reported in the

discussion, led to converse conclusions, i.e., K was not adequately defined to permit reliable conclusions, sediment has little or no effect on the flow, and the proportionality between ε_s and ε_m is equal to or less than unity.

The Bureau of Yards and Docks (B-2) published a technical publication in 1953 on dredging. This manual emphasizes that the character of material has a marked effect on the digging and pumping ability of the dredge. Also, the length of pipeline more or less governs the pipeline velocity, and a change in length will therefore have its effect on dredge output. It is pointed out that the economical pumping radius for any hydraulic dredge varies greatly with the class of material to be handled.

It is further stated that pipeline friction losses vary about as the square of the velocity. It also varies greatly with the character and weight of the materials being pumped. The Bureau recommends the use of $\frac{A_3V^2}{D}$ as the friction loss caused by moving a mixture through the pipeline. The factor A_3, which is a constant for any given material at a fixed velocity but changes with the change in character of the solids, is an inverse function of the velocity. V represents the pipeline velocity in fps, and D the inside diameter of the discharge line in inches. It is also noted that when solids are being carried, there is a critical velocity below which the friction loss increases very rapidly.

The dredge output in cubic yards of solids per hour can be computed from

$$C_T = \frac{A_2 V^a D^b}{J^c}$$

in which A_2 = a variable, depending on the character of the solids being pumped

V = pipeline velocity in fps

D = pipeline diameter in inches

J = the total head

a, b, and c = established constants

The correct determination of the factors A_3 and A_2 rests primarily on the judgment and experiences of the designer. No data is given for the constants a, b, and c.

Durand and Condolios (D-2) presented a very complete paper in 1953 on the hydraulic transport of sediments having a specific gravity of 2.65. Nine categories of sediment, from very fine sand to pebbles, were studied in four conduits of 40.6mm to 250mm diameters. Total sediment load varied from 50 to 600 grams per litre. Using a whistle meter for discharge measurements, it was found that for non-deposit regime and concentrations less than 20% by volume the manometer reading was not affected by the presence of sediment. Also, for fine sands in high velocity flows the presence of the material did not have an appreciable influence on the value of the head loss expressed in terms of

clear water, i.e., the presence of the material was not detectable on a metallic manometer. The head loss for muds was found to be the same as that for water in turbulent flow, if the head was expressed in height of mixture. For analysis of the fine sand and coarser material transport, it was found that the Gasterstadt relative head loss $(J - J_e)/J_e$ was a significant variable and related to the transport concentration by $(J - J_e)/J_e C_T = \theta$ where θ was to include V, D, d, etc. It was decided that for sediments having a sieve diameter d greater than 2mm, with a given velocity V, the head loss is independent of the sediment dimensions. The term J_e is the hydraulic gradient for the pipe transporting clear water under the same V as the corresponding J. In constructing θ, it was determined empirically that, for d and D a constant, $\theta = \theta(V)$. If D were varied, $\theta = \theta\ (V/\sqrt{gD})$. With both d and D variable, θ became

$$\theta = \theta\ ((V^2/\sqrt{gD})\ (\sqrt{gD}/w))$$

All those data for fine sand or coarser material transported entirely in suspension, by water, were correlated by

$$A_2\ (\sqrt{gD}/V)^3\ \ (w/\sqrt{gD})^{1.5}$$

where A_2 is an empirical constant.

The study also revealed the necessity for classifying the sediment mixture according to grain size into homogeneous

and heterogeneous classes. The complete classification suggested is:

a. Homogeneous mixtures: clays, fine ash, and very finely powdered coal (up to 20 to 30 microns).

b. Intermediary mixtures: silts (25 to 50 microns)

c. Heterogeneous mixtures:

1. Heterogeneous mixtures transported by suspension - fine sand, powdered coal, and slurry (from 50 microns to 0.2mm).
2. Transitions category: coarse sand and fine gravel coal (0.2mm to 2mm)
3. Heterogeneous mixtures transported by saltation; gravels, pebbles, and lumps of coal (above 2mm).

Chamberlain (C-10) 1955, studied the internal mechanism of sediment transport with 0.20mm sand sediment using 12 inch diameter smooth, helical corrugated, and standard corrugated pipes. Concentration profiles were taken along the vertical and horizontal diameters. Some of his conclusions were:

a. The Darcy-Weisbach resistance coefficient, for a given boundary was not significantly affected by the presence of fine sand until the total load of sediment increased to a magnitude which caused deposition to take place in the pipe.

b. The Darcy-Weisbach resistance coefficient

decreased with increasing Re and seemed to follow the Karman-Prandtl resistance equation for turbulent flow in smooth pipes, as long as all of the sediment was in suspension.

c. The mean velocity, at which deposition started, became less dependent on the magnitude of the total load as velocities were increased.

d. The horsepower input required to maintain a certain discharge of sand-water mixture was not materially greater than that necessary to pump the sand discharge of water, as long as all the sediment was in suspension. Horsepower was computed in terms of the discharge of mixture, with the unit weight and head loss expressed in feet of water.

e. Defining the point of most efficient operation as the minimum point of a constant total load curve on a J-V diagram and assuming operation at this point, helical corrugated pipe transported more sediment per unit time for a given horsepower than corrugated, and more than smooth pipe provided the total load was fairly small.

f. Helical and corrugated pipes delivered more sediment for a given mixture discharge than did smooth pipe. The smooth pipe required a higher mixture discharge than did the helical or plain corrugated pipes in order to maintain the same constant sediment discharge.

g. The horizontal concentration profiles were constant over a horizontal diameter of the smooth pipe.

h. The standard deviation of the sediment sieve

diameter affected the lower portion of the profile in the smooth pipes.

i. An absolute criterion for determining incipient deposition was presented. The criterion for incipient deposition is the maximum or rapid decrease (whichever is applicable) of a C_a/C_T versus C_T plot for a given distance above the bottom of a pipe and for a constant mean velocity.

j. $\mathcal{E}_s$ had a much different distribution, as a function of distance from the boundary, in a smooth pipe than in a corrugated pipe.

k. The Karman K in the Rouse number Z_1 was below 0.4 in smooth pipe. Concentration profiles deviated considerably from the Rouse equation.

l. The energy balance between the rate of energy dissipation required to maintain fine material in suspension and the decrease in the rate the fluid consumes energy seems to take place on the level of large scale energy transport eddies, and not to materially affect the small scale energy consuming turbulent eddies.

m. The difference, if any, between the energy required to transport a sediment laden fluid and an equal discharge of the homogeneous continuous phase is a function of the amount the sediment causes the piezometric head to deviate from a constant over any cross-section normal to the direction of the mean velocity.

Morris (M-5) 1955, proposed a new concept of flow in rough conduits based on tests conducted in 1954. Three basic types of flow: (1) isolated roughness flow, (2) wake-interference flow, and (3) skimming flow were postulated. Corrugated surfaces satisfy the wake interference type of flow, and the resistance coefficient f is given by

$$f = \left(\frac{1}{2 \log_{10} \frac{r_o}{2} + 1.75 + \frac{1}{\sqrt{2}} (2.5 - \xi) \frac{\beta \lambda}{r_o}} \right)^2$$

where β is a coefficient in $y = \beta\lambda$, y is the distance from the roughness crest to the breakpoint between the wall and the core velocity profiles. The length λ is the logitudinal spacing of the roughness elements and the r_o is the radius of the pipe axis to roughness crest. The dimensionless function ξ approaches 2.5 as β goes to zero with increasing velocity.

The resistance function

$$\frac{1}{\sqrt{f}} - 2 \log_{10} \frac{r_o}{\lambda}$$

will approach the same constant value 1.75 for all types of roughness elements, with increasing $\frac{Re\sqrt{f}}{\frac{r_o}{\lambda}}$. However, before this value of 1.75 is reached, the resistance coefficient increases with Re. It was also shown that Morris' equations can

be extended to surfaces of variable roughness by using the average values of the roughness dimensions.

Garde (G-1) 1956, published a master thesis which reports an extension of the testing undertaken by Chamberlain (C-10) at Colorado A & M. Garde's work presents a thorough study of the field of "Sediment Transport Through Pipes". His experiments were conducted with 0.60mm median diameter sediment flowing in water through 12 inch diameter smooth, helical corrugated, and standard corrugated pipes. The work of previous investigators is adapted and applied to his results. Garde summarizes as follows:

1. In the range of Re in which experiments were carried on, for clear water flow, the standard corrugated and helical corrugated boundaries behaved as hydrodynamically rough, while in the smooth boundary, the laminar sub-layer still covered the surface irregularities, and hence, it was hydrodynamically smooth.

2. For 0.60mm size sediment, presence of sediment always made the hydraulic gradient greater than that for clear water at the same velocity.

3. For helical corrugated pipe, the limit deposit velocity was a function of sediment concentration.

4. Helical corrugated pipe required nearly the same discharge of water sediment mixture to transport a given discharge of sediment of 0.60mm median size as sediment of 0.20mm median

size. For the same sediment discharge, standard corrugated pipe required more water discharge for 0.60mm size sediment than for 0.20mm size sediment. In the use of smooth pipe no conclusive results were obtained.

5. For each of the three boundaries - smooth, helical-corrugated, and standard corrugated - it was found that, for a given sediment discharge, increasing the size of the sediment resulted in greater energy requirements per foot of pipe per pound of sediment transported.

6. For a given size of sediment and a given sediment discharge, helical corrugated pipe required the least amount of energy per foot per pound of sediment transported; standard corrugated required the most energy.

7. Durand's equation did not fix the data collected from various sources, to an acceptable degree of satisfaction.

8. For reasons given in (1), f for clear water was constant with respect to Re for helical corrugated and for standard corrugated pipes. For smooth pipe f decreased with increase in Re and the Karman-Prandtl equation,

$$\frac{1}{\sqrt{f}} = 2 \log_{10} Re \sqrt{f} - 0.80$$

for turbulent flow in smooth pipes was found adequate to estimate f

9. In each of the three boundaries, f increased with increase in C_T, for a given Re. In both the suspended load regime and deposition regime, it was found that the equation

$$Re\sqrt{f} = \left[\frac{I}{\frac{d}{D}}\right]^{S_1} C_T^{1/3}$$

was adequate to give variation in f due to variation in d, D, and C_T for smooth pipes.

10. Sediment movement on the bottom of the pipe is quite comparable to the sediment movement on the bed of an alluvial channel.

Garde concluded that:

1. In the helical-corrugated boundary, the water-sediment discharge necessary to transport a given discharge of sediment is not affected by the size of the sediment from 0.2mm to 0.60mm diameter sediment.

2. Considering the aspect of energy requirement for transporting a given quantity of sediment, of a given size, helical corrugated pipe is the most economical.

3. The resistance coefficient f is adequate to study the flow of water-sediment mixture through all the three boundary forms studied in the regime of suspended load and in the regime of deposition.

4. The resistance coefficient f is affected by the sediment concentration and the characteristics of sediment and boundary form.

5. There is a need to investigate further the parameter relating the sediment size to the size of pipe; with

some ingenuity it may be possible to form a parameter different from

$$\left[\frac{I}{\frac{d}{D}} \right]^{S_1}$$

which will be more significant.

<u>Chien</u> (C-1) in 1956 presented an excellent paper which reviewed the status of sediment transport. The paper brought together in most of the experimental work and study that has been accomplished. The bibliography of this reference is an outstanding source of material.

While much of the paper deals with alluvial channel flow, parallel analogy can be drawn forth to apply to the closed channel conduit case of this thesis. Portions of Chien's paper are briefly set forth below.

<u>A.</u> <u>Grain</u> <u>Resistance</u>

The frictional resistance of sediment grains at the bed surface can best be described by the logarithmic formula based on the similarity theorem of von Karman. The average velocity, V, of the flow in the vertical direction may be given by the equation

$$\frac{V}{(R_b' \, Sq)^{\frac{1}{2}}} = \frac{2.3}{K} \log_{10} \left(12.27 \; x \; \frac{R_b'}{d} \right)$$

in which x is a corrective parameter for the transition from

smooth to rough and is itself a fraction of d/δ (δ being the thickness of the laminar sub-layer); x is unity for a hydraulically rough bed; K denotes the von Karman universal constant of turbulent exchange; and d is the representative grain size of the bed material. d has been variously recommended as the D_{65} and D_{90} size. In the above R_b' is the hydraulic radius of the grain resistance and S is the energy slope. To be noted is that transport is occurring both as suspended matter and as so-called "bed load" solid matter.

Of particular interest is the collection of data and discussion of the Karman constant K. In general, values are reported less than 0.4 ranging from 0.20 to 0.38. Chien states that a change in K means a change in velocity distribution. It can be shown that the theory of suspension yields only the concentration distribution of the suspended sediment, and the rate of suspended-load transport is determined by integrating the product of the local velocity and concentration from the surface of the bed layer up to the water surface. If velocity distribution changes, one should determine what causes the change and how the change can be predicted. The controversy over the possible reason for the change of K must be settled if further progress toward a general solution of the sediment transport problem is to be made.

From his discussion of the work of several authors, Chien concludes that (a) the sediment motion affects the velocity

distribution and (b) if the roughness changes K, the roughness element must be so pronounced that its height becomes comparable with depth of flow. The presence of the heavy fluid zone near the bed is very important in the following respects. When the turbulence generated at the bed in the heavy fluid zone enters the light fluid zone, the momentum exchange may become less effective due to the reduction of mass of the exchange flow. In the heavy fluid zone, part of the shear may also be transmitted to the boundary through the sediment particle in motion. These two effects, in combination with the decrease in turbulence level because of the energy spent in keeping sediment in suspension are all accounted for by a reduced value of K.

Chien also shows the result of a plot of the rate at which frictional energy is spent per unit weight of fluid and per unit of time in supporting the sediment in suspension versus K. Finally, a reduction of K from 0.4 to 0.2 does not mean that the average velocity of the sediment-laden flow will be twice that of the corresponding sediment-free flow. Evidently, the equation of von Karman given above is no longer adequate in describing a sediment-laden flow. The development of a new equation on velocity distribution must wait until the effect of heavy sediment concentration near the bed becomes better understood. It is believed, however, that the difference in average velocity between the sediment-laden flow and the sediment free flow will not be appreciable.

B. Sediment Transport Rate

Sediment has been divided into bed load and suspended

load according to the difference in the mode of movement and in the laws that govern its motion, and into bed-material load and wash load according to the difference in the source of material.

1. Suspended Load - Suspended load is the material moving in suspension in a fluid, being held up by the upward component of the turbulent current. Distribution of the relative concentration of suspended load is of the form set forth by Vanoni (V-1) and Rouse (R-9)

$$\frac{C}{C_a} = \left[\left(\frac{D_1 - y}{y} \right) \left(\frac{a}{D_1 - a} \right) \right]^Z$$

with $Z = \frac{w}{KV_o}$ (V_o = shear velocity at the wall). Hunt (H-12), taking into consideration the space occupied by the sediment particles, introduced a more exact distribution function as follows:

$$\left(\frac{C_y}{1 - C_y} \right) \left(\frac{1 - C_a}{C_a} \right) = \left\{ \left[\frac{1 - \frac{y}{D}}{1 - \frac{a}{D}} \right]^{\frac{1}{2}} \left[\frac{B_s - (1 - \frac{a}{D})^{\frac{1}{2}}}{B_s - (1 - \frac{y}{D})^{\frac{1}{2}}} \right] \right\}^Z$$

in which B_s is a constant that was found to be slightly smaller than unity. The above equations have been shown to describe the suspended-sediment distribution extremely well, except that the value of the exponent Z does not agree with Z_1, the exponent

that fits the measured data reported by Vanoni and others (V-1). Vanoni (V-1) and Ismail (I-1) proposed that

$$Z_1 = \frac{\omega}{\beta K V_o} = \frac{Z}{\beta}$$

and Hunt proposed

$$Z_1 = \frac{\omega}{B_s K_s V_o} = \frac{ZK}{B_s K_s}$$

No detailed explanation has been given as to what basically causes the difference between Z and Z_1 and what determines the value β or $B_s K_s/K$.

Einstein and Chien (E-4) revised the suspended load theory as follows:

a. The difference in sediment concentration at the vertical distance ℓ apart, ℓ symbolizing the mixing length of the turbulent flow, is expressed by $\ell \, dC_y/dy$ only for fine particles. For coarse particles the sediment distribution is highly skewed, and the higher deviating terms of the concentration C_y with respect to the vertical distance y have to be included.

b. It is unrealistic to assume that the turbulence eddy maintains its identity until it travels a certain distance, ℓ, and only then begins mixing. The model of the turbulence eddy is rather to be visualized as if the fluid carried by the eddy continuously mixes with the surrounding fluid on the course of the traverse of the eddy. However, the process of mixing is

much more active near the origin of the eddy than at points farther away. In other words, the mixing length is assumed to follow a certain probability distribution instead of possessing a finite value.

c. Because the turbulence is generated at the bed and dissipated toward the water surface, the characteristics of turbulence are not necessarily symmetric. In other words, for an upward flow and a downward flow originating from the same point, the mixing length and fluctuating velocity of the upward flow might not be the same as those of the downward flow.

Much work has been done in investigating the suspended load of fine particles, but very little has been done on the investigation of coarser materials. The suspended load theory breaks down near the bed where the particle size is of the same order of magnitude as the size of the eddies around the particle. The particle merely settles down between eddies without being lifted up again by the surrounding fluid masses. The thickness of the bed layer in which suspension becomes impossible has been found to be about two grain diameters. Within this layer, the particles are pushed forward by the flow. Their weight is no longer supported by the flow but by the bed itself, and the motion of the particles may be described as rolling, sliding, and jumping. This part of the load is usually called the bed load.

2. Bed Load - Much attention has been given in recent years to the transport of sediment via bed load movement. Chien

reviews several of the well-known formulae that have been developed. Full treatment of these formulae is beyond the scope of this thesis. However, it should be pointed out that practically all equations that have been developed for bed load theory are actually of compatible form and consist of the same parameters.

Evaluation of Total Load

The rate at which particles of a given size are picked up and placed in suspension is therefore, proportional to (a) the ralative number of these particles in the bed (b) the magnitude of the vertical turbulent velocity components which are present and capable of picking them up and (c) the percentage of time during which velocities exist that are capable of causing the fluid to pick up particles of that size.

The transport rate of suspended load is determined by integrating the product of local velocity and sediment concentration from bed layer up to the water surface. Knowing the rates of transport of suspended load and bed load, the total sediment discharge moving across the section is merely the sum of the two.

CHAPTER IV. CONSIDERATIONS AND ANALYSIS

In the writing of this thesis, a major effort has been directed toward the accumulation of data and written material through a reference survey. In this chapter, an effort is made to centralize and correlate the information obtained; and, where possible, focus it on the problem areas as set forth in the Introduction, Chapter I. It will be apparent to the reader that there is some divergence of opinion on the governing physical criteria concerning the transport of solids by pipeline. The author will attempt an "educated guess" or postulation where such is deemed appropriate. However, for the most part the data, analysis, and commentary are source material plainly referenced as such. Division of this chapter into sections is accomplished by setting forth the various design and/or investigation approaches to the analysis of pipeline flow with water-solids mixture.

Plots of Hydraulic Gradient, J versus Average Velocity V.

A study of J:V plots can provide information regarding both the resistance coefficient and the energy expended in maintaining the flow. An examination of Figure (5) shows how V and J vary with various concentrations and for three different boundary conditions. The slopes of each curve is two with the exception of that of the smooth pipe which is less than two. In the case of corrugated and helical corrugated

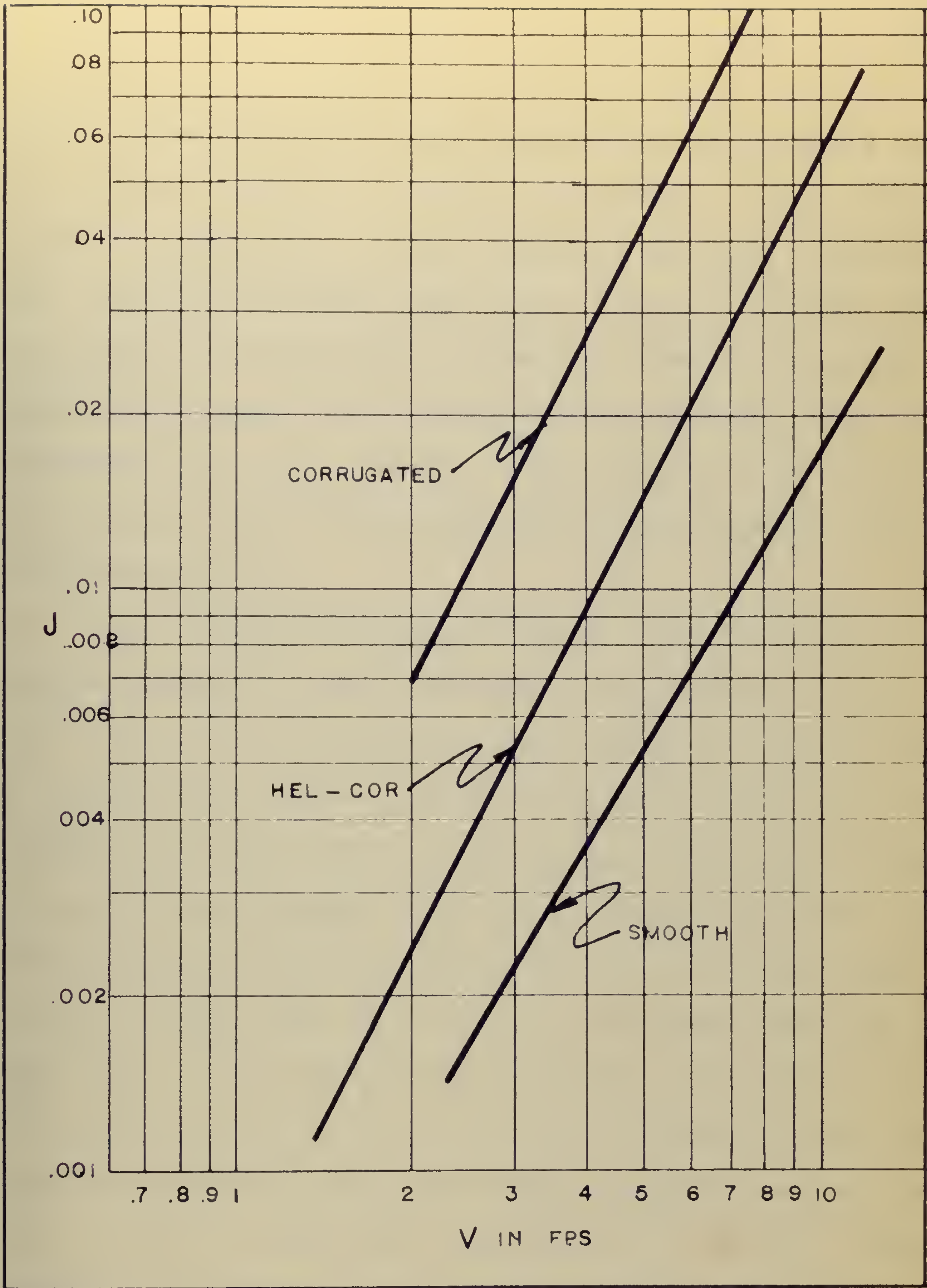

FIG.(5) – VARIATION OF HYDRAULIC GRADIENT WITH VELOCITY & CONCENTRATION (C-10)

pipe, hydrodynamically rough boundaries are provided and the so-called quadratic resistance law applies, i.e., the energy loss is proportional to the square of the mean velocity. In the case of the smooth pipe there is still a laminar sub-layer covering the protrusions so that the boundary is hydrodynamically smooth and the viscous effects are sufficiently pronounced to cause a slope of less than two. The first term of equation (17) is the difference, since equation (32) follows from (17).

In a pipeline flowing with pure water, the J:V plot will appear similar to the clear water line plotted in Figure (6) for a smooth circular pipe. When solids are added to provide a solids-water mixture, the J:V plot with curves of constant concentration superimposed thereon takes the form of Figure (6). A line AB can be drawn on the plot which delimits the zone of the regions with and without the deposit in the pipe. It is also evident that it is in close agreement with the minimum head loss. The velocity corresponding to the point at which the flow passes from one to the other region has been named the "economic velocity", the "limit deposit velocity", or the velocity of "incipient deposition " by various investigators. In terms of velocity, as velocity decreases, the lines of constant concentration show a decreasing hydraulic gradient. At a certain point, where the hydraulic gradient is a minimum, the constant concentration line goes back up. At a given

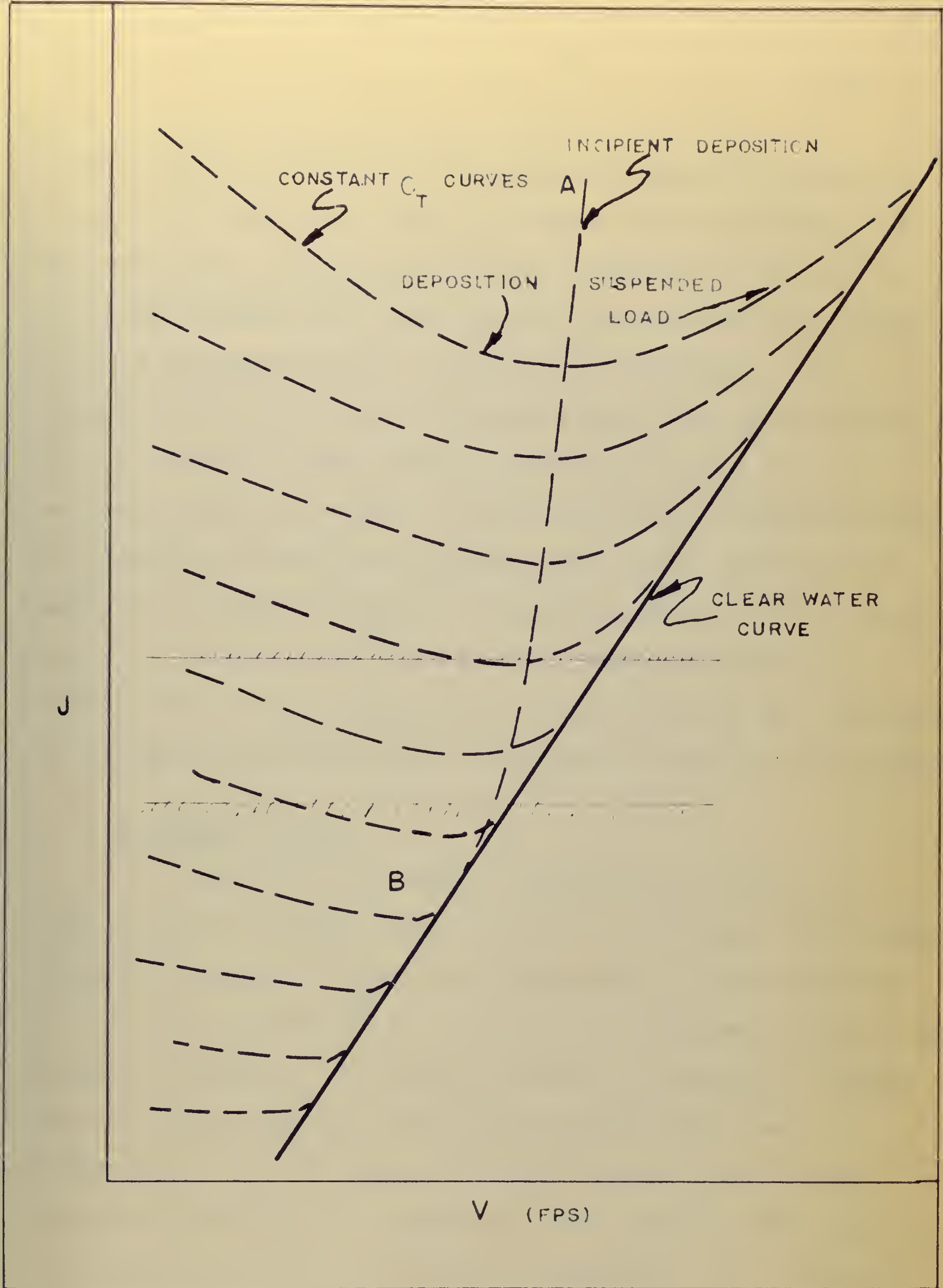

FIG.(6) — QUALITATIVE J-V DIAGRAM WITH CURVES OF CONSTANT TOTAL LOAD (C-10)

concentration, if the rate of flow is decreased, a point is reached at which the flow is no longer able to maintain all the solids in suspension and some of them are deposited in the pipe. Thus, due to deposition, the cross-section of the pipe is decreased and the velocity is increased. The resultant increase in the velocity permits the establishment of a new region, which owing to high velocity is able to maintain solids in motion. Over some range of discharge this equilibrium is maintained without much change in hydraulic gradient. If the quantity of discharge is further decreased, there is actually an increase in the hydraulic gradient. Chamberlain's (C-10) proposal of a criterion for the determination of incipient deposition will be outlined later in this chapter.

E versus W Plots.

Garde (G-1) presents a discussion of the E versus W plot in which, the parameter E represents the energy required per foot of pipe for transporting a unit weight of sediment per second and W is the rate of sediment transport in pounds per second. For sediment sizes of 0.60mm and 0.20mm and the three boundary conditions of Garde (G-1) and Chamberlain's (C-10) work, the curve for the larger size sediment lies above the smaller. This indicates that with an increase in the size of sediment transported, for a given W, E also increases. This may be accounted for by present

turbulent theory by arguing that for large-size sediment the small and large-scale eddies of the flow do not have sufficient energy to supply for transporting solids and therefore additional energy must be supplied. This points up the advantage of transporting solids in as fine a state as possible. Further analysis of the parameter E, provides additional information. Wilson (W-5) first suggested the use of the parameter to express the energy requirement. Since the energy required per foot of pipe is equal to $Q\gamma_m J$ and solids transported per second is equal to $\frac{Q\gamma_m C_T}{100}$ E will be

$$E = \frac{Q\gamma_m J}{\frac{Q\gamma_m C_T}{100}} = \frac{J}{\frac{C_T}{100}} \tag{49}$$

Therefore J must be a minimum in order to have E minimum, for the most economical transportation of sediment at a given concentration. (The point will correspond to the "limit deposit velocity".)

It is obvious that the general tendency of all E:W curves is for E to decrease as W increases. Therefore, it is more economical to transport a given volume of sediment at a large rate than to transport it at a small rate, as far as energy consumed per pound of sediment transported is concerned. For a given amount of sediment, E will be minimum when C_T is maximum, a study of the case where C_T is maximum is necessary.

For a given W, assuming γ_m is fairly constant, if Q is increased, then the corresponding C_T will decrease because

$$W \propto Q\gamma_m C_T \quad \text{and} \quad C_T \propto \frac{W}{Q\gamma_m} \tag{50}$$

Therefore, if Q is decreased, the concentration for a fixed W will increase. Furthermore, as the discharge is decreased more and more, E will reduce with a corresponding reduction in discharge. But there will certainly be a practical limit beyond which the discharge cannot be decreased in order to take advantage of economy in energy consumption per pound of sediment transported. With a decrease in discharge of water-sediment mixture, the section of pipe will be reduced due to deposition and, after a certain reduction of discharge it may be impossible to maintain a steady flow and the pipe may become clogged. Gardo (G-1) also shows that after reaching a minimum point, E will increase. For a given diameter of the pipe and a given size of sediment, this minimum value of E will depend on the value of W.

Durand's Equation

In 1953 Durand (D-2) gave an equation of the form of

$$\frac{J - J_e}{J_e C_1} = K_1 \left(\frac{\sqrt{gD}}{v}\right)^3 \left(\frac{1}{\sqrt{C_x}}\right)^{1.5} \tag{51}$$

where K_1 is a constant, C_1 is the relative absolute volume of

the sediment, and C_x is the drag coefficient defined by the equation

$$C_x = \frac{4}{3} \frac{gD}{\omega^2} \left(\frac{\rho_s - \rho_w}{\rho_w} \right) \qquad (52)$$

where ω is the settling velocity of particles. Garde (G-1) re-examined Durand's work and attempted correction of the limitation of the original work. A generally graded solids material was used since a sediment of rather uniform size, and both the deposition and non-deposition regions were studied. Results indicated the inadequacy of Durand's equation when applied to data taken under different and widely varying conditions.

Resistance Coefficient

The resistance coefficient is discussed from the view point of both clear water flow and sediment-laden flow.

A. Clear Water - In smooth pipe, f decreases as Re increases. It has been noted by several investigators that the Karman-Prandtl equation for turbulent flow in smooth pipes fits clear water reasonably well and hence the values of f can be estimated for smooth pipe using this equation.

$$\frac{1}{\sqrt{f}} = 2 \log_{10} Re \sqrt{f} - 0.80 \qquad (53)$$

B. Sediment-Laden Flow - Garde (G-1) found for

variation in the resistance coefficient f with Re, using C_T as the third variable and the flow being of the <u>suspended load region</u>, that for the size of sediment used, it was evident that the presence of sediment increased the resistance coefficient f. However, Chamberlain (C-10) using the same equipment but with a smaller diameter sediment, found that the values of f were not affected appreciably by concentration up to the point of incipient deposition. This means that the increase in the size of sediment necessitates additional energy to increase the eddy viscosity so that the larger sediment will be held in suspension. It may, therefore, be logically concluded that an increase in concentration will cause a corresponding increase in the resistance coefficient for a given Re. It also follows that for constant C_T, f decreases with an increase in Re. This may be explained by saying that with an increase in Re, the turbulence-creating eddies supply more and more energy to hold the sediment in suspension and therefore for the same concentration f should decrease with an increase in Re.

When the flow is in the region of <u>deposition</u>, the resistance coefficient is found to consist of the composite effect of various energy losses occurring in the pipe. These energy losses are due to (1) Boundary roughness (2) Turbulence (3) Sediment load carried in suspension (4) Moving sand dunes and (5) Increase in velocity as a result of decrease in cross-section caused by deposition. These effects are interdependent as, for example,

part of the boundary roughness becomes ineffective due to deposition. The problem, therefore, becomes very complicated making it extremely difficult to explain the effect of each of these factors on the value of f.

Use of Parameter Re $\sqrt{f}$.

It has been found that the relative thickness of the laminar sub-layer was inversely proportional to Re $\sqrt{f}$ in the case of clear water flow. Use has been made of this parameter in the study of resistance in alluvial channels and therefore, some investigators have felt that it might also play a significant role in sediment-water flow in pipes. A plot by Garde (G-1) of Re $\sqrt{f}$ versus C_T showed that the slope of the line for all available data was one third and furthermore, there was a tendency for the data to fall on this line regardless of the regime (suspended load or deposition) in which the data was collected. This relationship may be written as follows:

$$Re\sqrt{f} \propto C_T^{1/3} \tag{54}$$

or

$$C_T \propto (Re\sqrt{f})^3 \tag{55}$$

Thus it follows that the sediment load transported is proportional to the cube of the discharge when f is constant. When f is subject to small variation, this proportionality may still be used.

Garde (G-1) carries the above analysis further by assuming that the relative position of the lines on the $Re\sqrt{f}:C_T$ plot was a function of the parameter d/D and a plot was made with d/D as the ordinate and P the intercept on the $Re\sqrt{f}:C_T$ plot, as the abscissa, see Figure (7). The third variable on this plot was d, which gave a series of converging lines for constant values of d. The equations of each line on this d/D:P plot was obtained and the slopes and the intercepts were then plotted as functions of d. The slope relationship is represented by

$$S = 0.89d^{1/3} \tag{56}$$

The plot is shown in Figure (8). The resulting equation was

$$Re\sqrt{f} = \left[\frac{I}{\frac{d}{D}}\right]^{S_1} C_T^{1/3} \tag{57}$$

where $S_1 = \frac{1}{S}$ and I is the intercept obtained from Figure (8).

It is particularly interesting to note that three parameters have been evolved by the foregoing process. The first $Re\sqrt{f}$ describes the flow and may also be considered as $\frac{V_*D}{\nu}$, which is the shear velocity Reynolds number, or as the relative thickness of the laminar sub-layer. The second parameter C_T is the concentration of the total load. The third parameter, $\left[\frac{I}{\frac{d}{D}}\right]^{S_1}$ relates the sediment size to the size

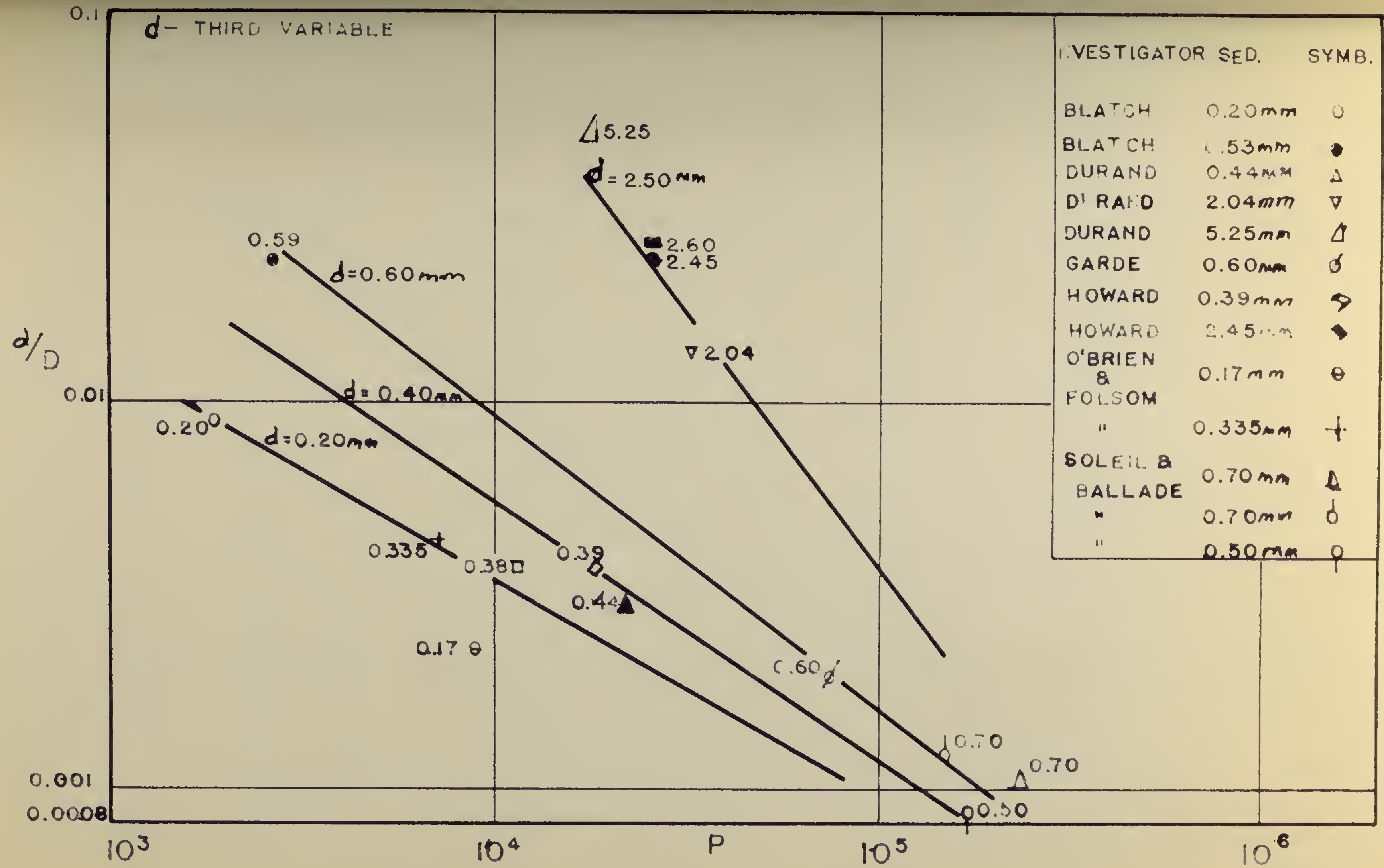

FIG. (7)— VARIATION OF P WITH d/D AND d (G-1)

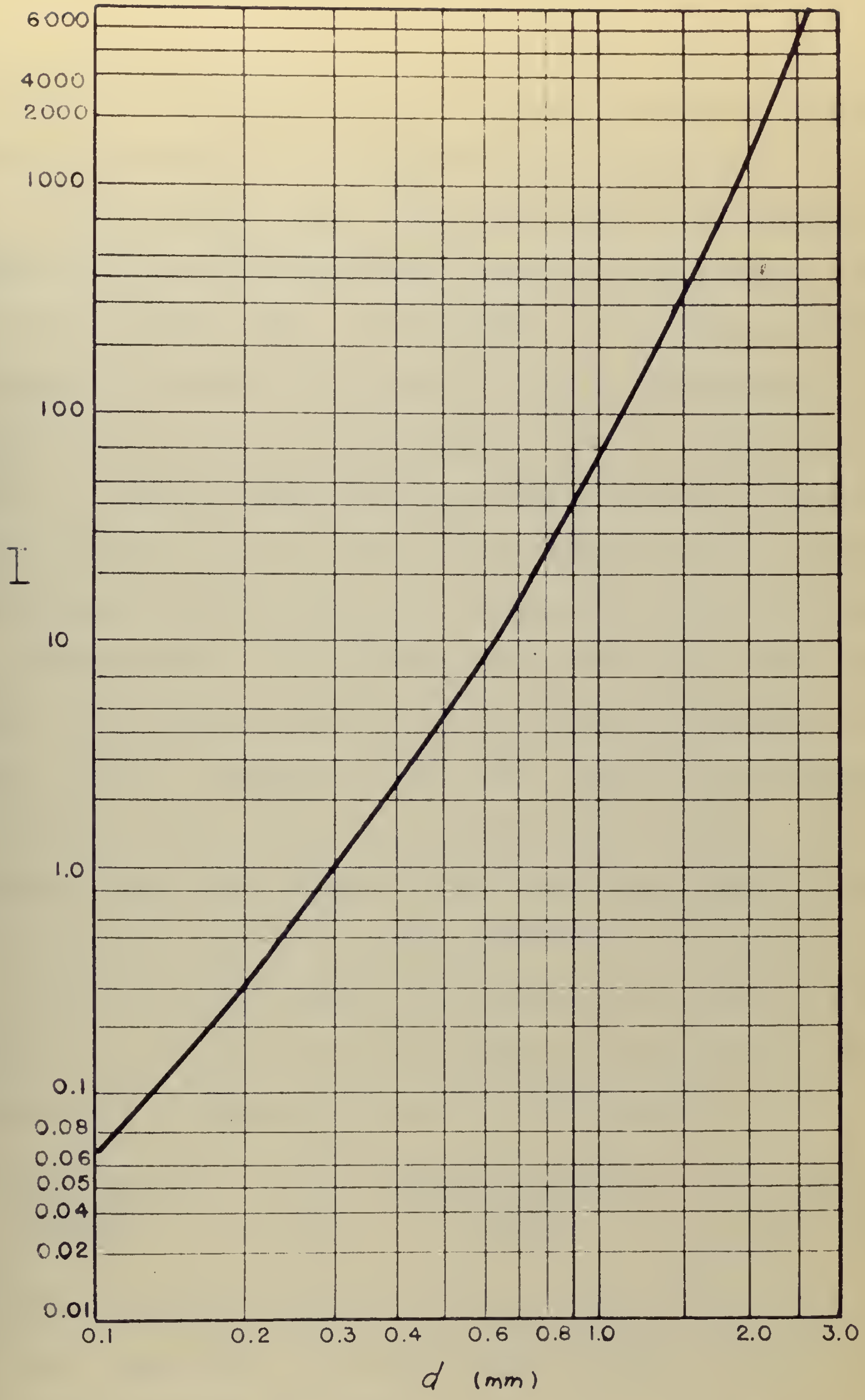

FIG. (8) — VARIATION OF 'I' WITH d (G-1)

of pipe and therefore may be considered as describing the relative geometry of the pipe and the sediment.

To verify the above equation, Garde (G-1) plotted all available data from different observers and varying conditions of flow, regime, etc. An excellent correlation obtained, as is shown in Figure (9), in spite of the difficulties in standarization of the data presented in the literature of this field. Certain limitations do obtain, however. When the slopes and intercepts of the lines in the d/D:P plot are plotted against the sediment size d, they are represented by relationships which are dimensional. To overcome this difficulty, Garde thought that the standard deviation of the size distribution of the sediment might provide a dimensionless term. However, the limited data available on the standard deviation did not help. It is also possible that the "length" term d might be a part of some sort of Richardson number or other parameter involving the difference in specific weight between the sediment and water. A second limitation is that any relationship between d/D, C_T, Re, and f should be a continuous function in the range.

Clear water regime $\longrightarrow$ Suspended load regime $\longrightarrow$ Deposition regime

The relationships given by equation (57) is not valid when C_T is zero, since when sediment is not present I, d/D, and S_1 are not defined and C_T is zero.

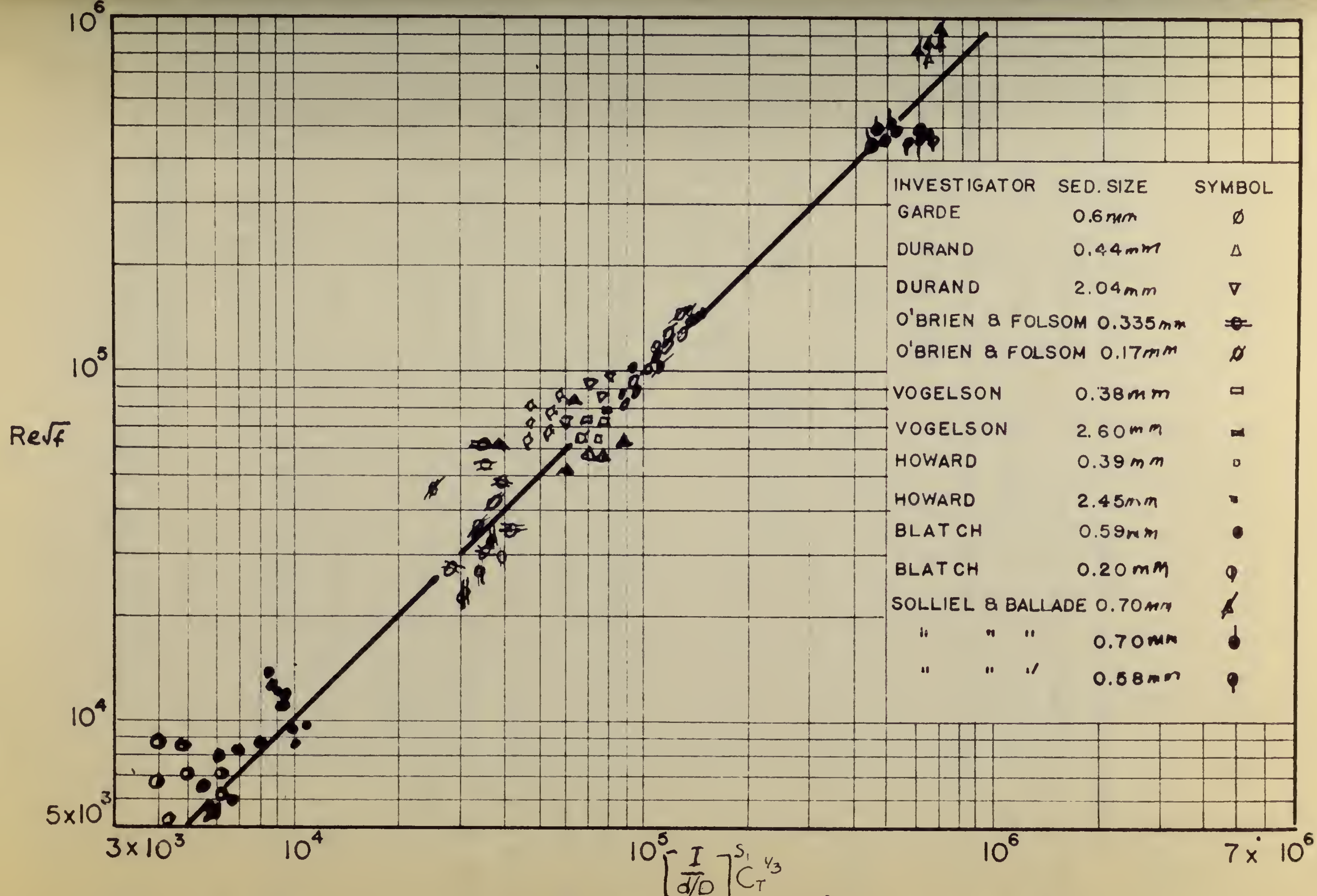

FIG. (9) – VARIATION OF $Re\sqrt{f}$ WITH $\left[\frac{I}{d/D}\right]^{S_1} C_T^{1/3}$ FOR SMOOTH BOUNDARY (G-1)

Garde suggests an expression of the form of:

$$f = \psi\,(Re,\ C_T,\ d/D\ ) \ = \ f_1 + f_2 \tag{58}$$

where f_1 = that part of f contributed by clear water

f_2 = that part of f contributed by inclusion of sediment.

To overcome the discrepancy for the present, Garde (G-1) suggests testing the value of f found under equation (57) against the value of C_T from

$$10^m \ = \ N\, C_T^{1/3} \tag{59}$$

where $m = \frac{1}{2} \left(\frac{1}{\sqrt{f}} + 0.8\right)$

and $N = \left[\frac{I}{\frac{d}{D}} \right]^{S_1}$

If C_T is less than this computed value, then the relationship of equation (57) is <u>not</u> valid as such a low concentration and f given by the Karman-Prandtl equation (53) should be used.

A third limitation to the use of equation (57) is that it was developed for a constant value of $\frac{\gamma_s}{\gamma_w}$ equal to 2.65.

In spite of these limitations, Garde believes that results can be obtained with 10 to 15% accuracy. Sample problems are worked out by Garde (G-1)

<u>Incipient Deposition</u>.

It can be concluded from the discussion on the J-V

curves that the minimum of a C_T constant curve on the J-V diagram is important in determining the minimum energy required for transporting a given total load. Figure (10) shows a plot of C_T versus V in which the dividing line between deposition and suspension is plotted. Chamberlain (C-10) indicates that this straight line is purely a matter of judgment, i.e., "an empirical conclusion in every sense of the word". Figure (11) results from extracting the limit velocities V_L from Figure (10) and plotting against C_T. Still further, if V_L is inserted in the parameter $V/\sqrt{gD}$, as $V_L/\sqrt{gD}$ then Figure (12) results. Referring to Figure (12) for a given boundary, the range of C_T for which an observer would probably say deposition was occurring is becoming larger and larger with increasing $V_L/\sqrt{gD}$. This has important ramifications as the horsepower (energy) will vary almost entirely with total load for large $V_L/\sqrt{gD}$ assuming operation corresponding to incipient deposition. The importance of this to a dredge operator is that, for a nearly constant mixture discharge, a large range of total loads is possible and still maintain operation near maximum efficiency. Variation of total load occurs when moving the suction line of a dredge from place to place. From the standpoint of sediment mechanics, once $V_L/\sqrt{gD}$ is increased and becomes less dependent on C_T, there is sufficient large scale eddy transfer energy available to accommodate a rather large range of total load.

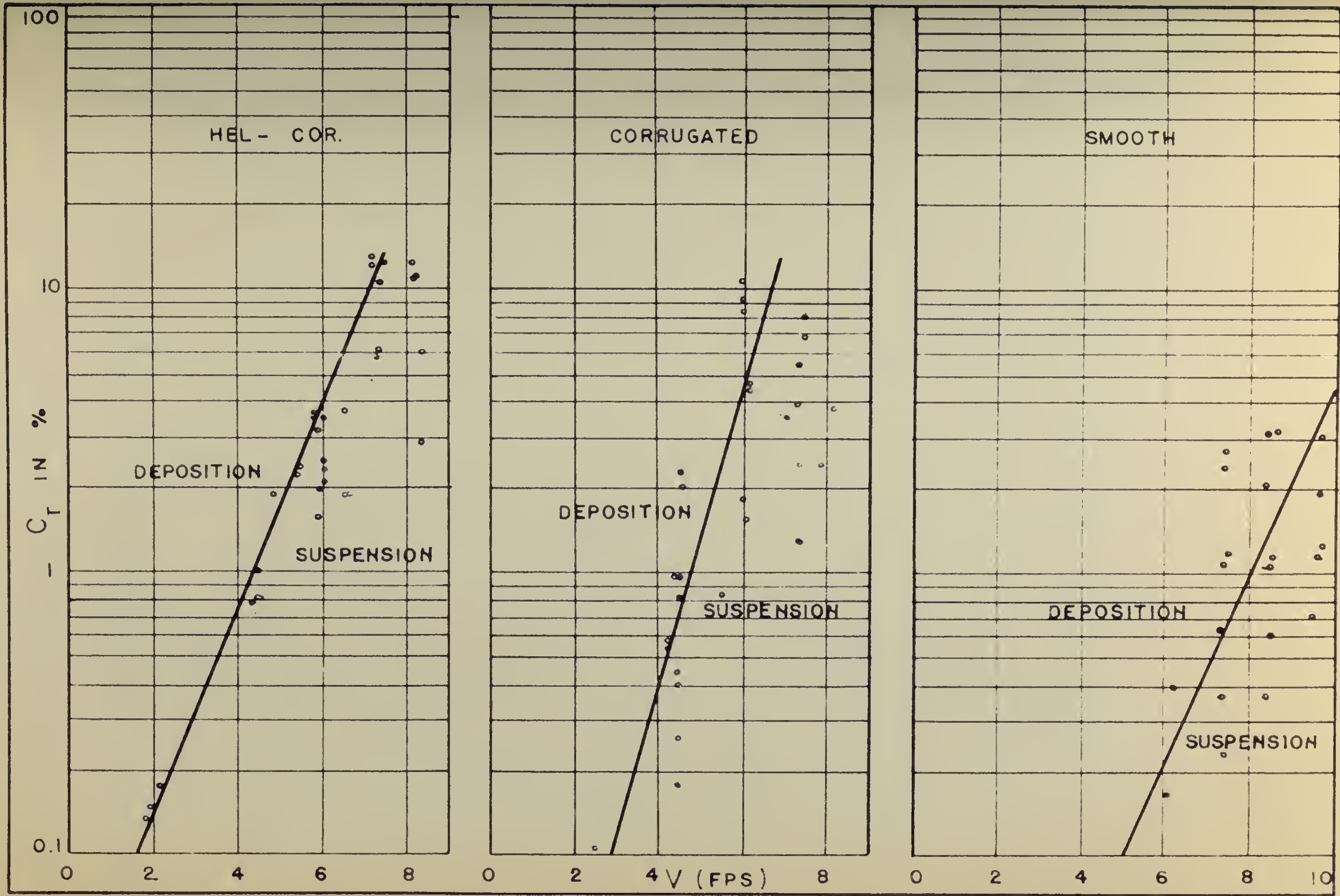

FIG.(10) — VARIATION OF TOTAL LOAD AT INCIPIENT DEPOSITION WITH VELOCITY

(C-10)

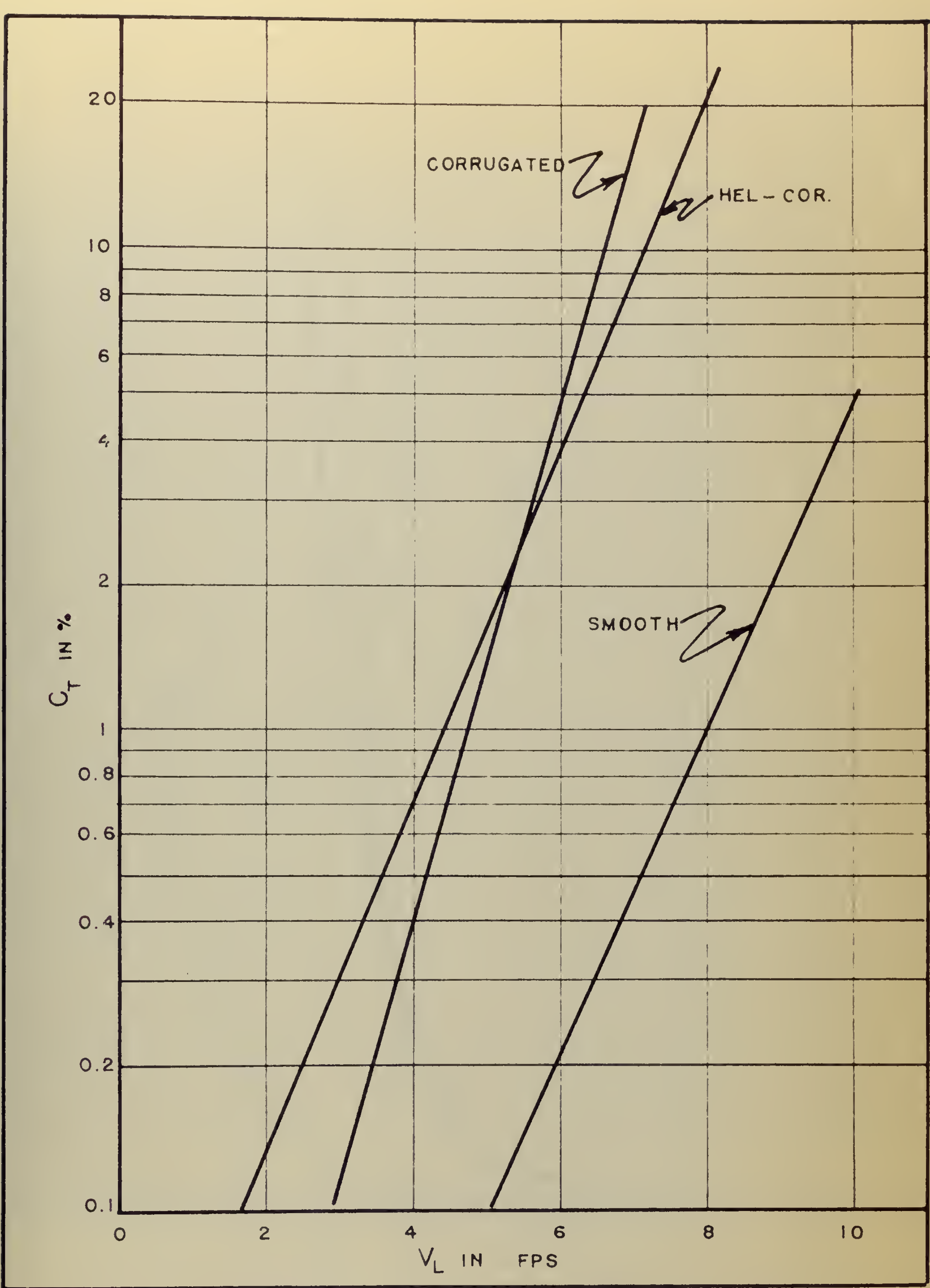

FIG. (11) — COMPARISON OF TOTAL LOAD CURVES AT INCIPIENT DEPOSITION (C-10)

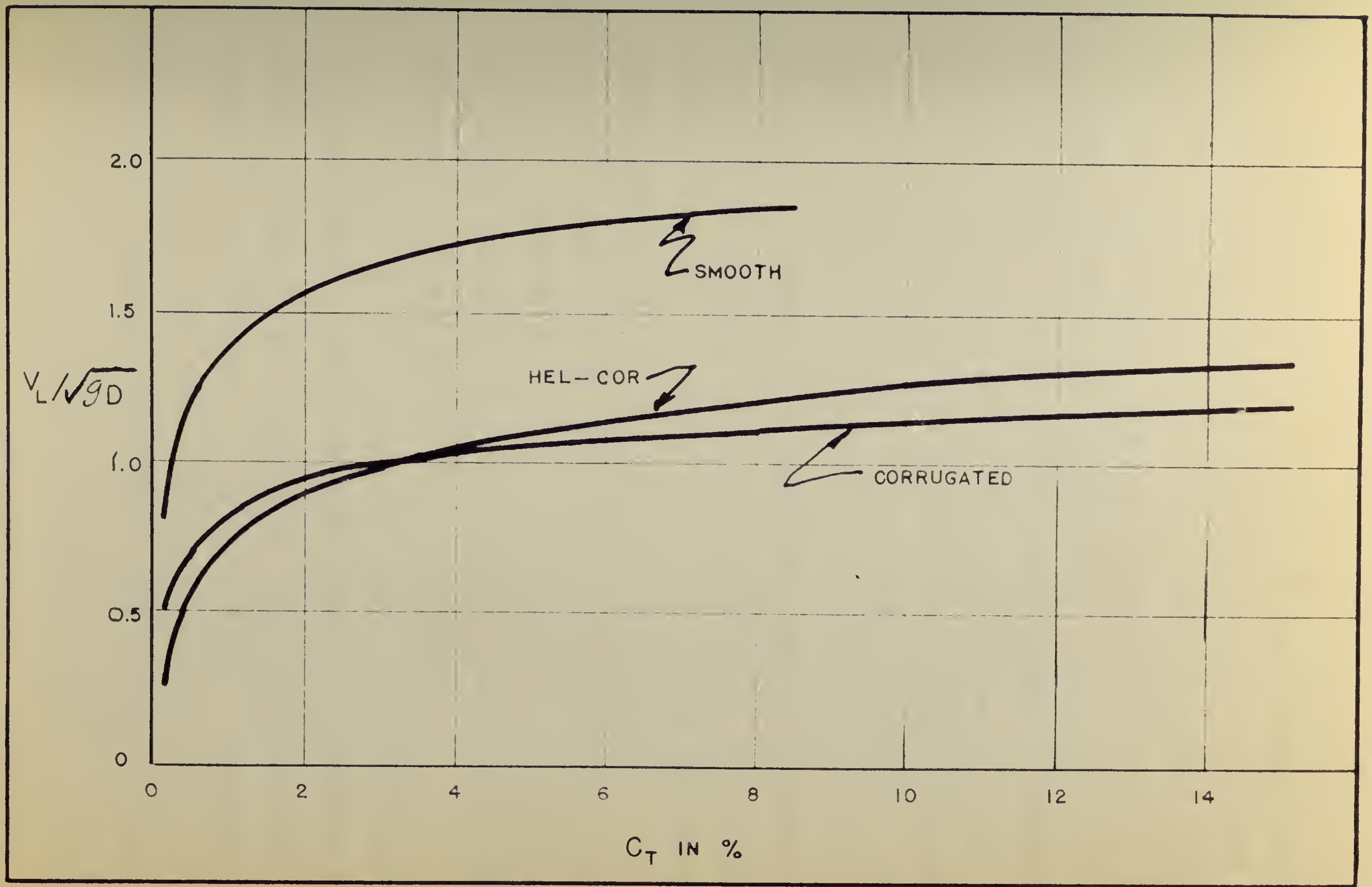

FIG. (12) — VARIATION OF LIMIT DEPOSIT VELOCITY PARAMETER WITH TOTAL LOAD (C-10)

Chamberlain (C-10) reports the establishment of an absolute criterion for incipient deposition. Consider Figure (13), a plot of C_T versus C/C_T at $y/D = 0.06$ for each $V/\sqrt{gd}$. Data in Chamberlain's study was insufficient for a complete analysis, but the trend is certainly evident and significant. The importance of the $V_L/\sqrt{gD}$ versus C_T plot in comparative studies of various pipes for design problems involving horsepower, rate of sediment discharge and excessive water losses points out the need for an evaluation of incipient deposition other than that of visual observation, which is not the same for any two observers, to determine incipient deposition. Chamberlain proposes that the maximum C/C_T , for a constant $V/\sqrt{gD}$ on a C/C_T versus C_T plot for a given y/D, be used in determining the C_T for incipient agradation.

The magnitude chosen for y/D is not important as long as it is consistent for a complete set of runs in a given pipe and picked somewhere near the bottom of the pipe so that appreciable differences in C/C_T are found.

Sufficient evidence has been presented by many investigators to prove that the most economical point for operating a sediment transport line is associated with incipient deposition. Figure (13) demonstrates that, as $V/\sqrt{gD}$ is increased, there was an increasingly large range of C_T over which an observer would say deposition was about to begin. The maximum becomes less and less distinct for increasing $V/\sqrt{gD}$.

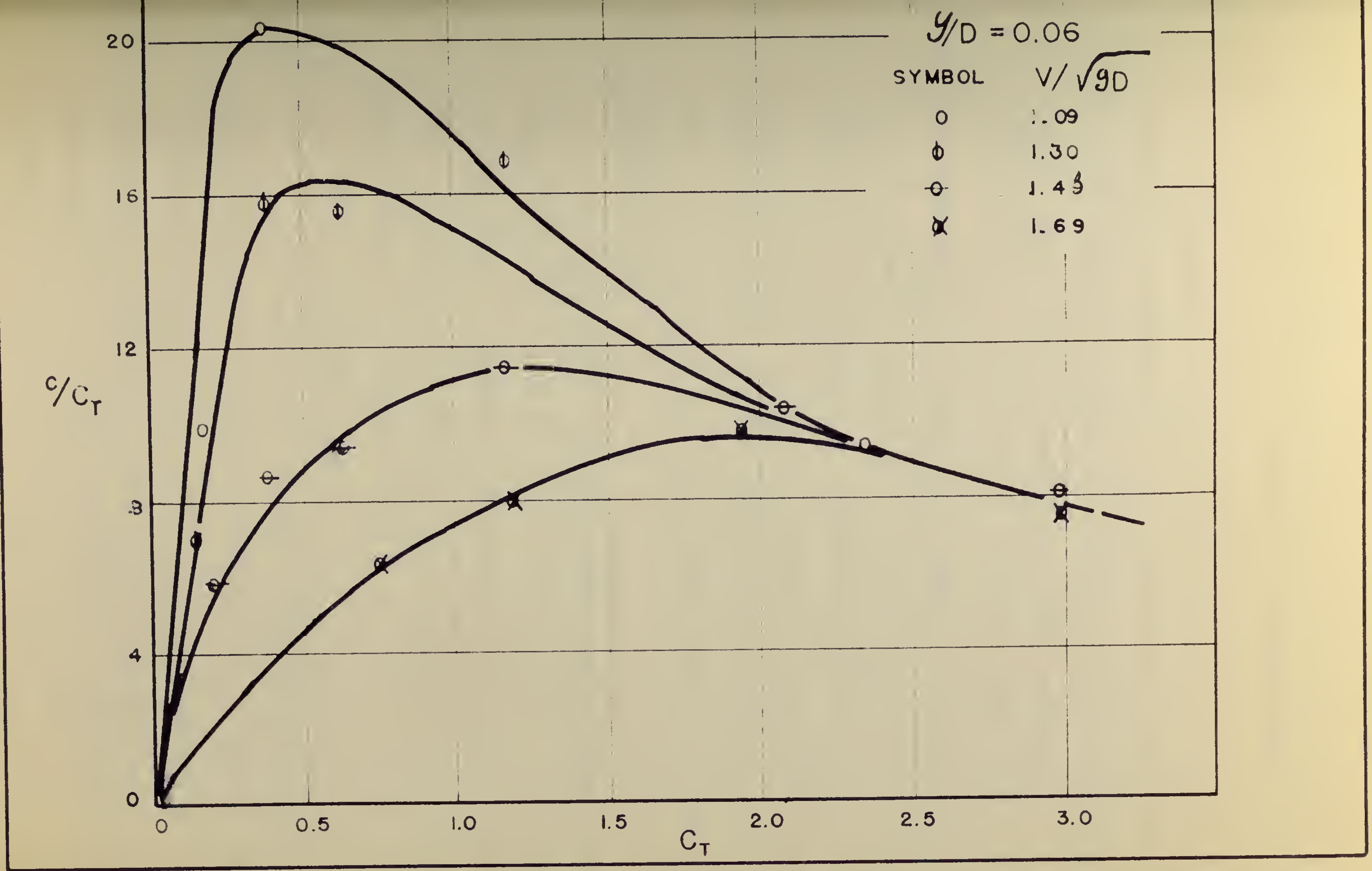

FIG. (13) – ABSOLUTE CRITERION FOR INCIPIENT DEPOSITION – 12" SMOOTH PIPE (C-10)

CHAPTER V. CONCLUSIONS

The general "rule of thumb" practice based on practical experience accumulated under actual operating conditions for dredge pipeline design and operation has established procedures which, while entirely empirical, have proven to be generally satisfactory. Much of this empirical data and procedure is well protected by individual firms as "trade secret information" and is closely guarded to prevent access by competing firms. Thus, it is difficult to examine publically the methods currently in use by dredge operators except in very minor detail. References (B-2) and (C-14) give as complete a summary of current methods as has been made available to the public in printed form.

The conclusions of the author are outlined below. The major opinion resulting from these is that a greater effort is required in research on pipeline two phase transport. The evaluation and application of theory to practice on large scale tests is urgently needed.

It is concluded that:

1. The most economic pipeline velocity is that corresponding to incipient deposition. Incipient deposition deserves further study. The absolute criterion for incipient deposition proposed by Chamberlain (C-10) needs further investigation. Accurate definition of the term is greatly lacking.

2. Friction loss seems best related to concentration rather than particle size.

3. No conclusion can be made as to the dividing line between colloidal and water-solids mix. There is little to commend study of this relationship as an important flow characteristic.

4. There is a need for extensive applied research with both full scale and model work to crystallize the physical relationships of the parameters concerned in two phase transport systems. The significant parameters in the study of this problem are J, ρ_s/ρ_w, d/D, C_T, V^2/gD, Re, $\sqrt{f}$ and Re$\sqrt{f}$. These parameters together with their combinations should be adequate to solve most transport problems.

5. Within the range of engineering accuracy required and based on data now available, an "apparent viscosity" for a given mixture may be used to provide estimates of flow characteristics.

6. Extensive experimentation and evaluation should be undertaken to secure further data on the von Karman, K, and the proportionality constant β, relating the momentum transfer coefficient, ε_m and the sediment transfer coefficient ε_s. The relationship elaborated on by Vanoni and Ismail relating the above to concentration represents the best theoretical contribution to an explanation of the sediment (solids) transport mechanism.

7. Published data on sediment transport in practically every case is based on sediment of a single size. Data is needed which covers the entire range of sediment mixture and which corresponds to actual dredging operations.

8. Percentage of solids transported by hydraulic dredge pipeline ranges up to 20% maximum and averages usually less than 15% depending on the character of the solids material. It would seem that this percentage could be increased to secure more economic efficiency.

9. Boundary conditions were not generally emphasized in this thesis. However, the studies by Chamberlain (C-10) and Garde (G-1) of helical corrugated pipe indicated that economies in transport operations can be had under certain conditions.

10. There is a need for study of the best means of instrumentation of dredge pipelines with suitably durable instruments for measurement of solids concentration, velocity, specific gravity, and other flow characteristics on an instantaneous continuous basis.

11. For a given diameter of pipe and sediment size, the minimum value of the energy input, E, will depend on the value of W, the rate of sediment transport. It is much more economical to transport a given volume of sediment at a large rate than a small rate as far as the energy consumed per pound of solids transported is concerned.

12. Durand's equation tends to be inadequate when applied to data taken under different and widely varying conditions.

13. Friction factor determinations are adequate to estimate quantities under two phase flow. The von Karman-Prandtl equation of turbulent flow can be used successfully and the relationship developed by Garde using the parameter $Re\sqrt{f}$ certainly has merit under the limiting conditions set forth by him. Further, verification on large scale operational dredge equipment is needed.

14. Dredge operators should make a concerted effort to provide for more freely exchanged information on practices and procedures. By such means the industry may more rapidly expand and achieve the transport economics that are potentially available.

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